[This question paper contains 2 printed pages.]

Your Roll No...............

K

Sr. No. of Question Paper : 5718

Unique Paper Code : 2372013501

Name of the Paper : Theory of Estimation

Name of the Course : **B.Sc. (Hons.), Statistics under NEP-UGCF**

Semester : V

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **three** questions from **Section I** and **three** questions from **Section II.**
3. Use of a simple calculator is allowed.

SECTION I

1. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a normal distribution with mean μ and variance σ^2. Show that $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator of σ^2.

   (b) Explain the concept of consistency. If $X_1, X_2, \ldots, X_n$ are random observations on a Bernoulli variate X taking the value 1 with probability p and the value 0 with probability $(1-p)$ then show that (i) $\bar{X}$ is a consistent estimator for p and (ii) $\frac{\sum_{i=1}^{n} X_i}{n}\left(1 - \frac{\sum_{i=1}^{n} X_i}{n}\right)$ is a consistent estimator for $p(1-p)$. (7,8)

2. (a) Let the random variable X is distributed as $P(\theta)$. Obtain an unbiased estimator for $e^{-(k+1)\theta}$. Comment on the result.

   (b) State Cramer-Rao Inequality. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population having pdf. $f(x,\theta) = \frac{1}{\pi[1+(x-\theta)^2]}, -\infty < x < \infty, -\infty < \theta < \infty$. Find the Cramer-Rao Lower bound of variance of an unbiased estimator of θ. Also, examine whether the MVB estimator exists for θ. (7,8)

3. (a) If T_n is a consistent estimator of θ, then T_n^2 is also a consistent estimator of θ^2.

   (b) Define Minimum Variance Unbiased Estimator (MVUE). Prove that an UMVUE is unique. (8, 7)

4. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0,\theta)$ population. Show that the largest order statistic $X_{(n)}$ is a complete sufficient statistic for θ.Using Lehman Scheffe's theorem, find the UMVUE of θ.

P.T.O.

(b) Suppose $X_1, X_2, \ldots, X_n$ be a random sample from the distribution having $pdf\, f(x,\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, 0 \le x < \infty, \theta > 0$. Show that the sample mean $\bar{X}$ is sufficient for θ and that $\frac{n-1}{n\bar{X}}$ is the only unbiased estimator of $\frac{1}{\theta}$ based on $\bar{X}$. (8, 7)

SECTION II

5. (a) Describe the method of moments. Let $X_1, X_2, \ldots, X_n$ are a random sample from the distribution with pdf. $f(x;\alpha,\beta) = \frac{\beta^n}{\Gamma\alpha}x^{n-1}e^{-\beta x}, x \ge 0, \alpha > 0, \beta > 0$. Estimate the parameters α and β by the method of moments.

(b) Find the maximum likelihood estimator of α and λ (λ is being large) of the distribution $f(x;\alpha,\lambda) = \frac{1}{\Gamma\alpha}.\left(\frac{\lambda}{\alpha}\right)^{\lambda} e^{-\frac{\lambda x}{\alpha}} x^{\lambda-1}, 0 < x < \infty, 0 < \lambda < \infty$. You may use the large values of λ, $\psi(\lambda) = \frac{d}{d\lambda}\log\Gamma\alpha = \log\, \lambda - \frac{1}{2\lambda}$ and $\psi'(\lambda) = \frac{d^2}{d\lambda^2}\log\Gamma\alpha = \frac{1}{\lambda} - \frac{1}{2\lambda^2}$. (7,8)

6. (a) What is a failure-censored sample in a life testing experiment? Obtain the maximum likelihood estimator of the expected lifetime and reliability function in case of failure censored sample from the following lifetime distribution: $f(x;\sigma) = \frac{1}{\sigma}e^{-\frac{x}{\sigma}}, x > 0, \sigma > 0$.

(b) Let X be binomially distributed with pmf $f(x,\theta) = \theta^x(1-\theta)^{1-x}, x = 0, 1$, where $\frac{1}{4} \le \theta \le \frac{3}{4}$. Show that the MLE of θ is $\frac{(2x+1)}{4}$. (8,7)

7. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu,\sigma^2)$, σ^2 is known. Obtain 100 $(1-\alpha)\%$ confidence interval for μ. If a random sample of size n=25 from a normal population with variance σ^2=144 has a mean of 65.3, construct a 95% confidence interval for the population mean μ.

(b) Explain the method of constructing confidence intervals for large samples. Find 100 $(1-\alpha)\%$ confidence interval for binomial proportion θ based on a random sample of size n for large samples. (7,8)

8. Describe any **three** of the following:
   a. Method of minimum Chi-square estimation
   b. Fisher-Neyman Criterion
   c. Properties of maximum likelihood estimator
   d. Regularity conditions for CRLB (15)