[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5707 **K**

Unique Paper Code : 2342013501

Name of the Paper : Algorithms and Advanced Data Structures

Name of the Course : **B.Sc. (H) Computer Science**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **two** sections **A** and **B**.
3. **Section A** carries **30** marks and each question in **Section B** carries **15** marks.
4. **Section A** is compulsory.
5. Attempt any **4** questions from **Section B**.
6. Parts of a question must be attempted together.

Section A

Q. 1 a) Differentiate between the crawling and indexing phases of a search engine's workflow. (3)

b) In the VH-tree, what is the maximum number of horizontal pointers that can be traversed consecutively on a path from the root to a leaf? Justify your answer based on the tree's fundamental property. (3)

c) Consider a binary counter of 4 bits that starts from 0 and supports only an INCREMENT operation. Using the accounting method of amortized analysis, determine the amortized cost per INCREMENT operation. Assign a credit to each bit flip and explain how these credits are used to pay for expensive operations. (4)

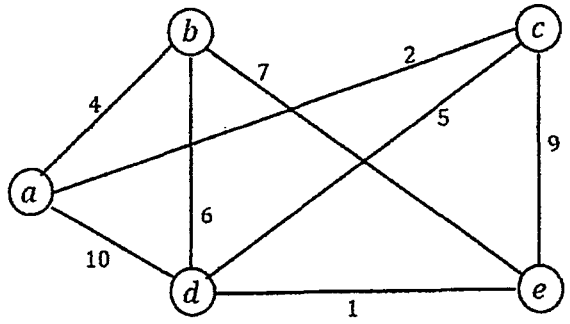
d) The Subset-Sum problem is defined as: Given a set of integers and a target sum T, does there exist a subset whose elements add up to T? Describe a brute-force algorithm for Subset-Sum and state its time complexity. Why does this algorithm fall in NP class? (5)

e) Give an example of a dataset consisting of 7 elements that results in the worst case for Quick Sort. Using this dataset, Sort the array using Quick Sort (with the last element as the pivot). Sort the same array using Randomized Quick Sort (selecting a random pivot in each partition). Compare the impact of pivot selection on the performance in both cases. (5)

f) What is meant by a residual graph G_f for a flow network G with a flow f and what is significance of a backward edge in this residual graph? Explain with the help of an example (5)

P.T.O.

g) Write Kruskal's algorithm for finding Minimum Spanning Tree. Apply the same on the following weighted undirected graph. Clearly show every step of addition of selected edge to generate MST.

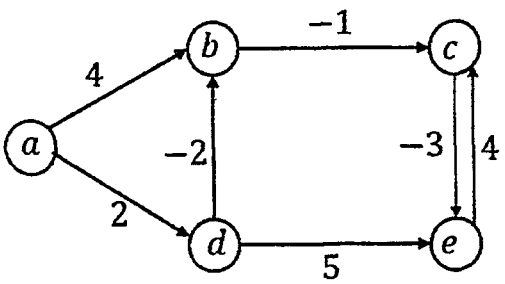


(5)

Section B

Q. 2 a) Explain Independent Set and Vertex-Cover problems. Prove that Independent Set $\leq_P$ Vertex-Cover. (7)

b) Apply Bellman-Ford algorithm to find the shortest path from a to all other vertices on the following weighted directed graph G. Show the distance progressions of all vertices after each iteration. When does Bellman-Ford algorithm fail? Justify your answer with proper example.



(8)

Q. 3 a) How the suffix trie of a string S of length n can be used to check if a pattern P of length m is a substring of S in $O(m)$ time? Illustrate with the pattern *"NAN"* on the suffix trie for *"BANANAS"*. Why is a suffix trie not the preferred data structure for large genomic sequences despite its powerful substring operations? (7)

b) Using Knuth-Morris-Pratt (KMP) algorithm, construct the failure function for $P =$ *"ABAABAC"* and find the first occurrence of the pattern P in the text string $T =$ *"ABABACABABABAABACABAC"* clearly illustrating all shifts and comparisons. Compare the Brute Force and KMP algorithms in terms of time complexity. (8)

Q. 4 a) Considering the set of words: {*programmer, programming, progress, project, python, slack, stack, stream, string, structure, algorithm, algebra, alpha*} make a standard trie to store these words. By converting it into a compressed trie, show how much space is saved. (7)

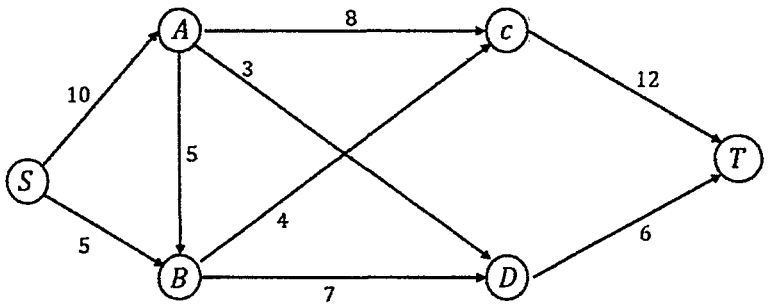
b) What is the minimum number of keys allowed in the root node, non-root internal node, and leaf node in a B-Tree of order m? Create a B-tree of order 3 with help of operations (insert 10), (insert 20), (insert 5), (delete 10), (insert 15), (insert 25), (delete 5), (insert 30). Show the state of tree after each operation. (8)

Q. 5 a) What is meant by a partition being helpful in Randomized-Select and prove that probability of a partition being helpful is at least $^1/_2$. Considering an unsorted array of integers $A = \{34, 17, 28, 12, 45, 19, 7, 72, 67, 4, 93, 52, 10, 86\}$, find 6^{th} smallest element using Randomized-Select algorithm. Element at $^1/_4{}^{th}$ location of resultant array is selected as pivot every time. (7)

b) Consider the Union-Find data structure where Union procedure performs union by rank and Find procedure performs path compression. Prove that irrespective of the sequence of operations, the rank of any node is strictly less than its parent. Prove that the maximum rank of any node in a connected component containing n nodes is at most $\lfloor log_2 n \rfloor$. How does the performance change if Union procedure does not perform union by rank? (8)

Q. 6 a) A queue is implemented using two stacks, Stack1 and Stack2. ENQUEUE pushes elements onto Stack1. DEQUEUE pops from Stack2 if it's not empty; if Stack2 is empty, all elements from Stack1 are popped and pushed onto Stack2 before the DEQUEUE. Using the accounting method, assign an amortized cost of 3 for ENQUEUE and 1 for DEQUEUE. Show that these credits suffice to pay for all operations. Explain why the amortized cost per operation is $O(1)$ even though some DEQUEUE operations may take $O(n)$ time. (7)

b) Apply the Ford-Fulkerson algorithm to find maximum flow from S (Source) to T (Sink) in the following graph G. Show the residual graph after every augmentation step. How does the Ford-Fulkerson algorithm ensure that the flow is maximized.

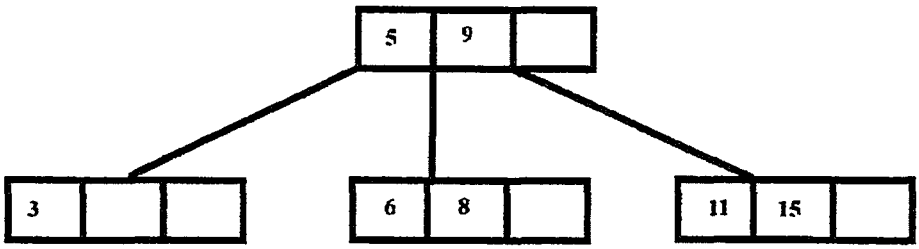


(8)

Q. 7 a) Describe how a node's height is determined during insertion in a typical skip list implementation. What is the probability that a node will reach a height of exactly k? Using this, show that the expected number of nodes at level h is $^n/_{2^h}$. Trace the search path for the element 42 in the following skip list. (7)

Level 3: -∞ ---------- 75 ---------- ∞
Level 2: -∞ --------- 15 ---------- 75 ---------- ∞
Level 1: -∞ --------- 15 ----- 25 ---------- 75 ---------- ∞
Level 0: -∞ --------- 15 ----- 25 ----- 42 ----- 75 ------ 90 --------- ∞

b) Explain the process of deletion of a key in a B-Tree. Consider a B-Tree of order 4 with following initial structure:



Delete 3 and 6 from the B-Tree and clearly show all the necessary steps involved in deleting these keys. (8)

(1500)