[This question paper contains 2 printed pages.]

Your Roll No...............

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Sr. No. of Question Paper	:	**9260**
Unique Paper Code	:	2353510001
Name of the Paper	:	DSE: Elementary Mathematical Analysis
Name of the Course	:	**B.Sc. (Physical Science and Mathematical Science)**
Semester	:	VI

Duration : 3 Hours   Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory. Attempt any **two** parts from each question. **All** questions carry equal marks. The use of the calculator is not allowed.

1. (a) Use the sequential criterion to prove that $\lim\limits_{x \to x_0} f(x)$ does not exist for the given function

$$f(x) = \begin{cases} 1, & if\ x\ is\ rational \\ 0, & if\ x\ is\ irrational \end{cases}, \quad x_0 \in R .$$

But the function $f(x) = \begin{cases} x, & if\ x\ is\ rational \\ -x, & if\ x\ is\ irrational \end{cases}$ has a limit at x_0 if and only if $x_0 = 0.$ (4,3.5)

(b) Using the definition of limit, prove that the following limit statements are true.

(i) $\lim\limits_{x \to 2}(4x - 5) = 3,$ (ii) $\lim\limits_{x \to 2} x^2 = 4.$ (3.5,4)

(c) Write the statement of the sequential criterion of continuity of a function f at x_0 and using the sequential criterion for the discontinuity of a function f at x_0, show that the signum function,

$$sgn(x) = \begin{cases} \frac{|x|}{x}, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases} \text{ is discontinuous at } x = 0.$$ (3,4.5)

2. (a) Use Intermediate value theorem to find a root of the equation

$$x^4 - x^3 - 10 = 0.$$ (7.5)

(b) Prove that the function $f(x) = \frac{1}{x}$ is continuous on $(0,1)$, but not uniformly continuous on $(0,1)$. (4,3.5)

(c) Define a uniformly continuous function on a set A and using the definition, show that the function $f(x) = x^3$ is uniformly continuous on $[0,3]$. (2.5,5)

3. (a) Show that the function $f(x) = x^3$ is integrable on the closed interval [0,1] and find $\int_0^1 f(x)dx$. (6,1.5)

(b) If f is monotone on $[a, b]$, then prove that f is integrable on $[a, b]$. Is the converse true? Justify your answer. (5,2.5)

(c) If f is any function defined and bounded on $[a, b]$, then both $\int_{\underline{a}}^{b} f(x)dx$ and $\int_{a}^{\overline{b}} f(x)dx$ exist and $\int_{\underline{a}}^{b} f(x)dx \leq \int_{a}^{\overline{b}} f(x)dx$. (7.5)

4. (a) If f is integrable on $[a, b]$. Then, prove that f^2 is also integrable on $[a, b]$. (7.5)

(b) If f is integrable on $[a, b]$ and $m \leq f(x) \leq M$ for all $x \in [a, b]$, then show that

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a).$$ (7.5)

(c) State the Fundamental Theorem of Calculus (First Form). Use it to calculate $\int_0^{\sqrt{3}} \sqrt{x}dx$. (2.5,5)

5. (a) Show that the series $\sum_{n=1}^{\infty} \frac{\sin(x+nx^2)}{n(n+1)}$ is uniformly convergent on **R**. (7.5)

(b) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n}{n^2} z^n$. (7.5)

(c) Establish the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} |x|^n$ in $-1 \leq x \leq 1$ by Abel's Test. (7.5)

6. (a) Define the radius of convergence of a power series, and hence find the radius of convergence of the series $\sum_{n=1}^{\infty} n^n z^n$. (7.5)

(b) Prove that the series $\sum f_n(x)$, where $f_n(x) = \frac{x}{1+nx^2}$, x being real, converges uniformly on any closed interval I using Weirstrass's M-test. (7.5)

(c) Show that every uniformly convergent series is pointwise convergent, but not conversely. Give an example. (5,2.5)

(100)