[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 9258 **J**

Unique Paper Code : 2354002004

Name of the Paper : Linear Programming

Name of the Course : **Mathematics : Generic Elective**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator is not allowed.

1. (a) Solve the following linear programming problem graphically :

Minimize $z = 5x_1 + 4x_2$

Subject to

$$3x_1 + 2x_2 \geq 40$$
$$7x_1 + 2x_2 \geq 14$$
$$3x_1 + 6x_2 \geq 12$$
$$x_1, x_2 \geq 0$$

(b) Find all basic feasible solutions of the following system of equations :

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$

(c) Show that the set $S = \{(x, y) \in R^2 \mid x^2 + 5x^2 \leq 8\}$ is a convex set.

2. (a) Solve the following linear programming problem :

Maximize $z = x_1 - 2x_2 + x_3$

Subject to

$$x_1 + 2x_2 + x_3 \leq 12$$
$$2x_1 + x_2 - x_3 \leq 6$$
$$-x_1 + 3x_2 \leq 9$$
$$x_1, x_2, x_3 \geq 0$$

P.T.O.

(b) Solve the following linear programming problem by the Simplex method :
Minimize $z = 3x_1 - x_2$
Subject to

$$x_1 - 3x_2 \geq -3$$
$$2x_1 + x_2 \geq -2$$
$$2x_1 + x_2 \leq 8$$
$$x_1, x_2 \geq 0$$

(c) Using Simplex method, solve the following linear programming problem and show that it has an unbounded solution :
Minimize $z = x_3 - x_4$
Subject to

$$x_1 - 3x_4 + x_5 = 1$$
$$x_2 + 6x_4 - 5x_5 = 6$$
$$x_3 - 3x_4 + 2x_5 = 5$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

3. (a) Using Simplex method, solve the following linear programming problem and show that it has no solution :
Minimize $z = -x_1 - 3x_2 + x_3$
Subject to

$$x_1 + x_2 + 2x_3 \leq 4$$
$$-x_1 + x_3 \geq 4$$
$$x_3 \geq 3$$
$$x_1, x_2, x_3 \geq 0$$

(b) Write the dual of the following linear programming problem :
Minimize $z = 4x_1 + 4x_2 + x_3$
Subject to

$$x_1 + x_2 + x_3 \leq 2$$
$$2x_1 + x_2 = 3$$
$$2x_1 + x_2 + 3x_3 \geq 3$$
$x_1 \geq 0$, x_2 unrestricted in sign, $x_3 \leq 0$

(c) Show that the Dual of Dual of MAX-problem is the MAX-problem itself.

4. (a) Obtain an initial basic feasible solution for the following cost minimization transportation problem using

(i) North West Comer Rule

(ii) Least Cost Method

	P	Q	R	S	Supply
A	14	87	48	27	71
B	52	35	21	81	47
C	99	20	71	63	95
Demand	71	35	47	60	

(b) Obtain an initial basic feasible solution for the following cost minimization transportation problem using Vogel's Approximation Method and then find the optimal solution by MODI method.

	P	Q	R	S	Supply
A	9	7	12	8	18
B	15	14	12	15	4
C	10	9	6	12	6
D	14	12	11	12	12
Demand	6	14	15	5	

(c) Solve the following cost minimization Assignment Problem of assigning Jobs to Machines :

Jobs

Machines	J_1	J_2	J_3	J_4	J_5
M_1	6	9	5	7	4
M_2	3	8	4	5	7
M_3	2	7	3	4	6
M_4	5	6	5	7	5

5. (a) Solve the following assignment problem to maximize the total profit.

Jobs

Machines	J_1	J_2	J_3	J_4	J_5
A	8	2	2	5	4
B	10	9	2	8	4
C	5	4	9	6	1
D	3	6	2	8	7
E	5	6	10	4	3

(b) Explain the maximin and minimax principle used in theory of games and state any three major assumptions of the game theory.

(c) Solve the following game by using maximin (minimax) principle, whose payoff matrix are given below :

Player A	Player B			
	B_1	B_2	B_3	B_4
A_1	1	5	3	4
A_2	5	6	4	5
A_3	6	7	0	3

Include in your answer (i) strategy selection for each player, (ii) the value of game to each player. Does the game have saddle point?

6. (a) A Company management and the labour union are negotiating a new three-year settlement. Each of these has four strategies :

I : Hard and aggressive bargaining
II : Reasoning and Logical approach
III : Legalistic strategy
IV : Conciliatory approach

The cost of the company is given for every pair of strategy choice.

Union Strategies	Company strategies			
	I	II	III	IV
I	20	15	12	35
II	25	14	8	10
III	40	2	10	5
IV	-5	4	11	0

What strategy will the two sides adopt? Also determine the value of game using saddle point criteria.

(b) Use dominance rule to reduce the size of the following payoff matrix and hence, find the optimal strategies and value of the game.

Player A	Player B		
	B_1	B_2	B_3
A_1	1	7	3
A_2	5	6	4
A_3	7	2	0

(c) For the following payoff tables, transform the zero-sum game into an equivalent linear programming problem

Player A	Player B		
	I	II	III
I	20	15	12
II	25	14	8
III	40	2	10

(100)