[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 9256 **J**

Unique Paper Code : 2353012004

Name of the Paper : Biomathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. Each part of questions is of **7.5** marks.

1. (a) From the data given below showing US population from year 1800 to 1860 :

Year	Population (in millions)
1800	5.3
1810	7.2
1820	9.6
1830	12.9
1840	17.1
1850	23.2
1860	31.4

i. Find the best value of $k = (P_{n+1} - P_n)/P_n$ and $r = \ln(P_{n+1}) - \ln(P_n)$, where P_n represents the population at the end of n^{th}decade, for the discrete model and continuous model of population for growth respectively.

ii. Use continuous model to predict for population in year 3000. Do you expect the continuous model to remain accurate in predicting the long-term growth behavior of the population?

(b) Consider the following population model:

$$P_{n+1} = \frac{2P_n}{\left(1 + \frac{P_n}{K}\right)},$$

P.T.O.

Where K is carrying capacity. Find its equilibrium points and discuss their nature. What can be said if $0 < P(0) < K$, where $P(0)$ is the initial population.

(c) The rate of increase of bacteria in a culture is proportional to the number present. The population multiplies by a factor n in the time interval t. The population is found to increase by 2455 from $t = 2$ to $t = 3$ and by 4314 from $t = 4$ to $t = 5$. If initial population is p_0 then show that $p_0 = 4291$ approximately and that when $t = 3n$ is about 2.33.

2. (a) State Law of mass action. Derive mathematical model for concentrations of X and Y in the following chemical reaction

$$A + X \rightarrow 2Y$$

with rate constant k. Assuming initial concentrations of X and Y are X_0 and Y_0 respectively and A is held at a fixed concentration say a. Solve the system to obtain $X(t)$ and $Y(t)$.

(b) Derive Volterra-Lotka model for Predator - Prey interaction; and find conditions for existence of steady state.

(c) Suppose a drug is administered at regular time intervals say $t = 0, t_0, 2t_0, \ldots$ and each dose raises drug concentration by fixed amount c_0. Find expression for maximum concentration c_m and $r = \lim r_n$ as n tends to ∞, where r_n is residue after n doses at $t = nt_0$. Also show $c_m = c_0 + r$.

3. (a) Assuming that the population with size N is divided into three nonintersecting groups—the group of those who have the disease and can infect others (I), the group of those who do not have the disease and can be infected (S) and the group of those who recovered with permanent immunity (R). Write the SIR model for infectious diseases along with properly defining all the parameters used; and mention the conditions for which infectives will increasing or decreasing. Also determine the average number of secondary infections produced by the single infective among susceptible present initially.

(b) In the SIS Model for infectious diseases, show that if $\alpha N - \beta > 0$ for infection rate α and per capita rate of recovery β, then the rate of change for the group of infectives (I) is given by a logistic equation. What happens if $\alpha N - \beta < 0$? . Explain the long-term behavior of the disease.

(c) Define equilibrium states for the two-component system,

$$\frac{dx}{dt} = f(x,y) \quad and \quad \frac{dy}{dt} = g(x,y).$$

Also, define when such equilibrium points are stable.

If $x \geq 0$ and $y \geq 0$, find the equilibrium states for the system:

$$\frac{dx}{dt} = y^2 - y + x - 1 \quad and \quad \frac{dy}{dt} = y - x.$$

Also, discuss if these points are asymptotically stable or not.

4. **(a)** Sketch the trajectories in the phase plane of the system

$$\frac{dx}{dt} = 5x + 2y$$
$$\frac{dy}{dt} = 2x + 2y.$$

(b) For the predator-prey model

$$\frac{dV}{dt} = \frac{2}{3}V - \frac{V^2}{6} - \frac{VO}{1+V}$$
$$\frac{dO}{dt} = \delta O\left(1 - \frac{O}{V}\right),$$

Obtain the coordinates of the equilibrium point A which is the intersection point of the null cline $O = V$ and $O = \frac{2}{3}\left(1 - \frac{V}{4}\right)(1 + V)$. Discuss the stability of the equilibrium point A. Show that the region

$$R : 0 < V < 4 \,;\, 0 < O < 4$$

is a basin of attraction and the given system have a periodic solution.

(c) State the Limit Cycle Criterion for the system

$$\frac{d^2x}{dt^2} + x = -\varepsilon g\left(x, \frac{dx}{dt}\right).$$

Also establish the condition whether the system tends to a limit cycle when $x = B\cos(t)$.

P.T.O.

5. (a) Show that

$$\frac{dx}{dt} = x(y-1),$$
$$\frac{dy}{dt} = \mu - y(x+1),$$

has an equilibrium point, which is a stable node, for $\mu < 1$, that becomes a saddle-point as μ passes through the bifurcation point $\mu = 1$. Also show that there is an additional equilibrium point when $\mu > 1$, which is a stable node.

(b) Define:

(i) Bifurcation

(ii) Bifurcation point

(iii) Bifurcation diagram

Make sketches for Pitchfork bifurcation, Saddle-node bifurcation and Hopf bifurcation.

(c) Discuss the stability of limit cycle when $P_0, P_1, P_2, \ldots$ are the points on Poincare line and P_n is at a distance of s_n form P_0 and such that $s_{n+1} = f(s_n)$, where P_0 is the fixed point.

6. (a) Show that the iteration scheme

$$x_{n+1} = 1 - \mu x_n(1 - x_n)$$

has a stable fixed point $x_0 = 1$ for $\mu < 1$ and that $\mu = 1$ is a bifurcation point where the fixed point $x_0 = \frac{1}{\mu}$ appears. Show that the periodic doubling occurs as soon as μ exceeds 3.

(b) Let $M(\alpha_1)$ be the Jukes-Cantor matrix with parameter α_1, and $M(\alpha_2)$ the Jukes-Cantor matrix with parameter α_2. Compute $M(\alpha_1)M(\alpha_2)$ to show it has the form $M(\alpha_3)$. Give a formula for α_3 in terms of α_1 and α_2.

(c) Explain Kimura 2-parameter and 3-parameter models along with their corresponding distance formulas. Write the expression of the log-det distance between S_0 and S_1.