This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5439**

Unique Paper Code : **2374001201**

Name of the Paper : **Introductory Probability**

Name of the Course : **Generic Elective - Statistics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section A is compulsory.

Attempt any *five* questions from Section B.

Use of non-programmable scientific calculator is allowed.

Section-A

1. Answer the following : 1×5,2×5

(*a*) If X and Y are independent random variables, then E(XY) = ... and cov(X, Y) = ...

(*b*) If P(A) = 0.59, P(B) = 0.30 and $P(A \cap B) = 0.21$, find $P(A' \cap B')$.

(*c*) Define *r*th moment about the mean.

(*d*) State classical definition of probability.

(*e*) Let X_1 and X_2 are independent random variables so that $X_1 \sim P(5)$ and $X_2 \sim P(2.5)$. What will be the distribution of $X_1 + X_2$?

P.T.O.

(*f*) If X and Y are independent random variables with variances 3 and 5 respectively, find variance of $2X + 3Y$.

(*g*) If a random variable X is distributed as a Binomial distribution with mean 4 and variance 3, find its mode.

(*h*) The moment generating function of a random variable X is $M_X(t)$. Find the moment generating function of $Z = \frac{X-\mu}{\sigma}$, stating clearly the properties used.

(*i*) Let X_1 and X_2 are independent random variables so that $X_1 \sim N(5, 25)$ and $X_2 \sim N(2.5, 9)$. What will be the distribution of $X_1 - 2X_2$?

(*j*) State the Central Limit Theorem.

Section-B

2. (*a*) A rare but serious disease has been found in 0.01 percent of a certain population. A test has been developed that will be positive for 98 percent of those who have the disease and be positive for only 3 percent of those who do not have the disease. Find the probability that a person tested as positive does not have the disease.

(*b*) An urn contains 6 white, 4 red and 9 black balls. If 3 balls are drawn at random, find the probability that :

(*i*) two of the balls drawn are white,

(*ii*) one is of each colour,

(*iii*) none is red,

(*iv*) at least one is white. 7,8

3. (a) Two socks are selected at random and removed in succession from a drawer containing five brown socks and three green socks. List the elements of the sample space, the corresponding probabilities, and the corresponding value w of the random variable W, where W is the number of brown socks selected. Also, find the distribution function of the random variable W.

(b) The diameter X of an electric cable, is assumed to be a continuous random variable with p.d.f.

$$f(x) = 6x(1 - x),\ 0 \le x \le 1$$

(i) Check whether $f(x)$ is a p.d.f. or not.

(ii) Determine the number k such that $P(X < k) = P(X > k)$.

(iii) Compute $P\left(X \le \frac{1}{2} \middle| \frac{1}{3} \le X \le \frac{2}{3}\right)$. 8,7

4. (a) Given the following table :

x	$p(x)$
0	1/125
1	12/125
2	48/125
3	64/125

Compute $E(X)$, $E(X^2)$ and $E(5X - 1)^2$.

(b) Given that X has the p.d.f. $f(x) = \frac{1}{8}\binom{3}{x}$ for $x = 0, 1, 2$ and 3, find the moment generating function of this random variable and use it to determine μ'_1 and μ'_2. 8,7

P.T.O.

5. (*a*) Define the negative binomial distribution. Find its mean and variance. Show that geometric distribution is special case of negative binomial distribution.

(*b*) If 2 percent of the books bound at a certain bindery have defective bindings, use the Poisson approximation to the binomial distribution to determine the probability that 5 of 400 books bound by this bindery will have defective bindings ? 8,7

6. (*a*) Suppose a random variable X is distributed as a binomial variate with parameters n and p. Write the probability mass function of X. What is the distribution of $Y = n - X$?

(*b*) A taxi cab company has 12 Ambassadors and 8 Fiats. If 5 of these taxi cabs are in the shop for repairs and Ambassador is as likely to be in for repairs as a Fiat. What is the probability that :

(*i*) 3 of them are Ambassadors and 2 are Fiats ?

(*ii*) at least 3 of them are Ambassadors ? 8,7

7. (*a*) Define Normal distribution function. Also prove that linear combination of independent normal variates is also a normal variate.

(*b*) Let X_1 and X_2 be two independent random variables having geometric distribution $q^k p$; $k = 0, 1, 2, \ldots\ldots$ Show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform. 8,7

8. (*a*) Define Gamma and Exponential Distribution. Also obtain mean and variance of exponential distribution.

(*b*) The mean yield for one-acre plot is 662 kilos with a standard deviation 32 kilos. Assuming normal distribution, how many one-acre plots in a batch of 1.000 plots would you expect to have yield (*i*) over 700 kilos, (*ii*) below 650 kilos, and (*iii*) what is the lowest yield of the best 100 plots ? 8,7