(ii) Draw Hasse Diagram.

(iii) Find the maximal and minimal elements in A.

***

[This question paper contains 8 printed pages]

Your Roll No. : .....

Sl. No. of Q. Paper : **5796 I**

Unique Paper Code : 2342011202

Name of the Paper : Discrete Mathematical Structures

Name of the Course : **B.Sc. (H) Computer Science (NEP – UGCF-2022)**

Semester : II

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Question No. **1** is compulsory.

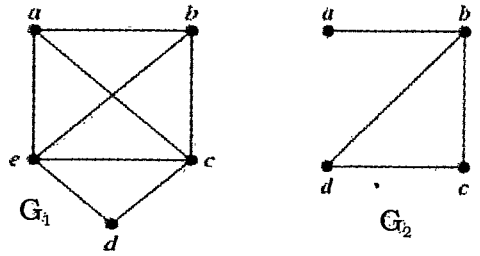
(c) Attempt any **4** of Questions Nos. 2 to 7.

(d) Parts of a question must be answered together.

(e) Use of a simple calculator is allowed.

Section - A

1. (a) Define a Hamilton path and Hamilton circuit. List a Hamilton path or Hamilton circuit in the graphs G1 and G2 given below if it exists. 5



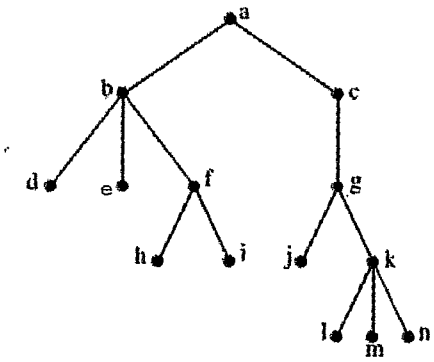
(b) (i) Find the contrapositive, the converse and the inverse of the statement : *"The home team wins whenever it is raining".* 3

(ii) Show that if n is an integer and 3n + 2 is odd, then n is odd using a proof by contraposition. 2

(c) Find the greatest common divisor of 414 and 662 using the Euclidean algorithm. 5

(d) Let f be the function from the set X = {2, 3, 4, 5, 6, 7} into the set Y = {0,1, 2, 3, 4} defined by $f(x) = 2x \pmod 5$. Write f as a set of ordered pairs. Is f one – one or onto ? Justify. 5

7. (a) How many leaves does a full 4-ary tree with101 vertices have ? Given the rooted m-ary tree below, answer the following questions : 7



(i) Draw the subtree that is rooted at g ?

(ii) Which vertices are ancestors of k ?

(iii) What is the value of m for the given rooted m-ary tree ?

(iv) Is the tree a balanced tree ? Give reason for your answer.

(b) Given a relation R on set A = {3, 5, 9, 15, 24, 45} such that : 8

$R = \{(a, b) : a \text{ is divisor of } b \text{ and } a, b \in A\}$

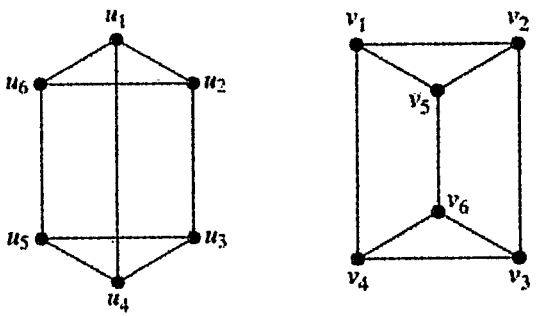
(i) Show that R is a POSET.

(b) Consider the following premises : 8

"It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip", and "If we take a canoe trip, then we will be home by sunset".

Determine whether the conclusion *"We will be home by sunset."* follows logically from the above set of premises.

6. (a) What do you mean by graph invariant ? Is the following pair of graphs isomorphic ? Justify. 7



(b) Evaluate 3^{644} mod 645 using Fast Modular Exponentiation algorithm. 8

(e) (i) How many distinct 5 – letter passwords can be formed from English alphabets if repetition is allowed and at least one letter must be a vowel ? 3

(ii) How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points ? 2

(f) Let 'a' be a numeric function such that : 5

$$a_r = \begin{cases} 1 & 0 \le r \le 3 \\ 2^r + 3 & r \ge 4 \end{cases}$$

Find ∇a and $S^{-2}\, a$.

Section - B

2. (a) In how many ways can three examinations be scheduled in a seven–day week if more than one examination can be scheduled on the same day. 3

(b) Let R be a relation on the set of real numbers such that 4

$R = \{(a, b) \mid a^2 + b^2 = 1\}$

State whether the relation R is reflexive, symmetric, antisymmetric and transitive. Briefly justify your answer in each case.

(c) Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3 for every integer $n \geq 1$. 8

3. (a) Let N (x) be the statement "X has visited North Dakota", where the domain consists of the students in your school. Express each of these quantifications in English. 3

(i) $\exists x N(x)$

(ii) $\forall x N(x)$

(b) Find the sum and the product of $(1001)_2$ and $(1101)_2$ without using decimal number system. 4

(c) In a survey of 60 people, it was found that 25 people read Hindi newspaper, 26 read English newspaper, 26 read Urdu newspaper, 9 read both Hindi and Urdu newspaper, 11 read both Hindi and English newspaper, 8 read both English and Urdu newspaper, 3 read all three newspapers. Find : 8

(i) The number of people who read at least one of the three newspapers.

(ii) The number of people who read exactly one newspaper.

(iii) The number of people who read both Hindi and Urdu newspaper, but not English ?

4. (a) (i) Define planar graph. Is K_5 a planar graph ? Justify your answer. 5

(ii) Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have ? 2

(b) Let 'a' and 'b' be two numeric functions defined as follows : 8

$$a_r = \begin{cases} 2 & 0 \leq r \leq 2 \\ 2^{-r} + 5 & r \geq 3 \end{cases}$$

$$b_r = \begin{cases} 3 - 2^{-r} & 0 \leq r \leq 1 \\ r + 2 & r \geq 2 \end{cases}$$

Determine the sum and product of numeric functions a and b.

5. (a) In how many ways can a committee of 5 members be formed from 11 Senators so that : 7

(i) Senator A is always included

(ii) Senator A is always excluded

(iii) At least one of senator A and senator B will be included