$y > 0$, $z > 0$.

(b) Generate seven random Numbers using Linear Congruence Method $x_{i+1} \equiv (ax_i + b) mod\ (m)$ where $m = 10$, $a = 1$ *and* $b = 7$, seed no $x_0 = 115$

(c) Given the following data.

x	2	3	4
y	-3	2	-1

Set up the residuals to fit the model $y = cx^2$ to the data set Using the sum of Absolute Deviations Criterion (not solve for c).

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6908 **J**

Unique Paper Code : 2354000010

Name of the Paper : Introduction to Mathematical Modelling (GE)

Name of the Course : **B.Sc.(H)/B.A.(H)/B.Com.(H)/BMS**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt **any two** parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable Scientific Calculator is allowed.

1. (a) In a fish farm, fish are harvested at a constant rate of 2100 fish per week. The per- capita death rate for the fish is 0.2 fish per day per fish, and the per-capita birth rate is 0.7 fish per day per fish.

(i) Write down the word equation describing the rate of change of fish population. Hence obtain a differential equation for the number of fish.

(ii) Determine if there are any values for which the fish population is in equilibrium.

(b) A tank contains 100 litres of salt water. Initially S_0, kg. of salt is dissolved. Salt water flows into the tank at the rate of10 litres per minute, and the concentration C_{in} (t) (kg. of salt/litre) of this incoming water-salt mixture varies with time. Salt

x	1	2	3
y	2	5	8

(b) Fit the curve y = b + ax by using Least Square Method where

x	1	2	3	4	5
y	1	1	2	2	4

(c) Use the Middle Square Methods to generate 12 random numbers using

$x_0 = 13$ (seed no).

6. (a) Using Monte Carlo Simulation write an algorithm to calculate that part of volume of the sphere $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant, $x > 0$,

4. (a) Develop a model with three differential equations describing a predator-prey interaction, where there are two different species of prey and one species of predator.

(b) Write down the compartment battle model of blue army B(t) and red army R(t), also the differential equations following the balance law.

(c) Determine the Predator and prey model in limiting cases of prey with no predator and predator with no prey.

5. (a) Fit the model $y = ax$ from the following data. Express the three residuals r_1, r_2 *and* r_3 corresponding to the data points using Chabyshev Criterion. (Not solve for c)

solution flows out at the same rate at which it flows in that is volume is constant. Find the differential equation for the amount of salt in the tank at time t. (Note that concentration can be defined as the mass of salt per unit volume of mixture.)

(c) Solve the system of differential equations:

$$\frac{dx}{dt} = -k_1 x, \; x(0) = x_0$$
$$\frac{dx}{dt} = k_1 x - k_2 y, \; y(0) = 0, \; k_1 \neq k_2$$

Where k_1, $k_2 > 0$ determine the rate at which a drug, antihistamine or decongestant moves between two compartments in the body, the GI-tract and the bloodstream, when patient takes a single pill.

Here $x(t)$ is the level of drug in the GI-tract and $y(t)$ is the level in the bloodstream at time t.

2. (a) In a population, the initial population is $x_0 = 100$. Suppose a population can be modelled using the differential equation $\frac{dX}{dt} = 0.2X - 0.001X^2$ with an initial population size of $x_0 = 100$ and a time step of one month.

(b) Write the balance law and find the differential equation for the lake pollution model. And find the solution of that differential equation.

(c) Suppose that a mineral formed in an ancient cataclysm-perhaps the formation of the earth itself originally contained the uranium isotope ^{238}U (which has a half-life of 4.51×10^9 years) but no lead, the end product of the radioactive decay of uranium isotope ^{238}U. If today the ratio of ^{238}U atoms to lead atoms in the minerals body is 0.9 when did the cataclysm occur?

3. (a) Consider a disease where all those infected remain contagious for life (they never recover). Ignore any births and deaths. Develop a pair of differential equations for the number of susceptibles S and number of infectives I.

(b) Construct a compartmental diagram for the model and develop appropriate word equation for the rates of change of recovered and susceptibles.

(c) Let $X(t)$ denote the number of prey per unit area and $Y(t)$ the number of predators per unit area. Formulate differential equations and compartment model for the prey and predator densities.