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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5377**

Unique Paper Code : **2354001202**

Name of the Paper : **Introduction to Linear Algebra**

Name of the Course : **Common Prog Group**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

1. (*a*) Let x and y be vectors in $\mathbf{R}^n$. Prove that $\|x + y\| \leq \|x\| + \|y\|$.

(*b*) Solve the following system of linear equations using Gauss-Elimination method :

$$x_1 + 2x_2 + 3x_3 = 1$$
$$x_1 + 3x_2 + 5x_3 = 2$$
$$2x_1 + 5x_2 + 9x_3 = 3.$$

(*c*) Solve the following system of linear equations using Gauss-Jordon method :

$$2x_1 + 3x_2 - x_3 = 9$$
$$x_1 + x_2 + x_3 = 9$$
$$3x_1 - x_2 - x_3 = -1.$$

P.T.O.

2. (a) Define rank of a matrix. Find the rank of the following matrix :

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}.$$

(b) Find the characteristic polynomial of the matrix :

$$A = \begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix}.$$

Also, find all the eigenvalues and eigenspace for any *one* eigenvalue.

(c) Is the matrix :

$$A = \begin{bmatrix} -3 & -1 & -2 \\ -2 & 16 & -18 \\ 2 & 9 & -7 \end{bmatrix}$$

diagonalizable ? Justify.

3. (a) Show that the set $\mathbf{R}^2$, with the usual scalar multiplication but with vector addition given by $[x, y] \oplus [w, z] = [x + y, 0]$ is not a vector space.

(b) Use Simplified Span Method to find a simplified general form for all the vectors in Span(S), where :

$S = [x^3 - 1, x^2 - x, x - 1]$ is the subset of $\mathbf{P}_3$.

(c) Show that the subset $\{[-1, 2, -3], [3, 1, 4], [2, -1, 6]\}$ of $\mathbf{R}^3$ forms a basis of $\mathbf{R}^3$.

4. (*a*) Prove or disprove that the subset :

$$S = \left\{ \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix}; a, b \in \mathbf{R} \right\}$$

is a subspace of M_{22} under usual matrix operations.

(*b*) Use Independent Test Method to find whether the set :

$$S = \{[2, -1, 3], [4, -1, 6], [-2, 0, 2]\}$$

of vectors is linearly independent or linearly dependent.

(*c*) Determine whether the vector X = [7, 1, 18] is in the row space of the matrix :

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}.$$

If so, then express [7, 1, 18] as a linear combination of the rows of A.

5. (*a*) Define the linear transformation. Determine whether the following function is a linear transformation :

$$T_1 : M_{32} \to P_4 \text{ given by } T_1 \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} = a_{11}x^4 - a_{21}x^2 + a_{31}.$$

(*b*) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear operator that performs a counterclockwise through an angle 30°. Find the matrix representation for T with respect to the ordered basis B = {[1, 1], [2, 3]} for R^2.

P.T.O.

(c) Let $T : P_2(\mathbf{R}) \to P_3(\mathbf{R})$ be given by :

$$T(p(x)) = \int p(x)\,dx.$$

Find the matrix for T with respect to standard bases for $P_2(\mathbf{R})$ and $P_3(\mathbf{R})$. Use this matrix to calculate $T(3x^2 - 2x + 5)$ by matrix multiplication.

6. (a) Suppose $L : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear operator with :

$$L([1, 2]) = [1, 1] \text{ and } L([2, 0]) = [2, -4].$$

Give a formula for $L([x, y])$ for any $[x, y] \in \mathbf{R}^2$. Also, find a basis for Ker(L) and Range(L).

(b) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear operator given by :

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find a basis for Ker(T) and a basis for Range(T). Also, verify that :

$$\dim(\text{Ker}(T)) + \dim(\text{Range}(T)) = \dim(\mathbf{R}^3).$$

(c) Let $T : M_{23}(\mathbf{R}) + M_{22}(\mathbf{R})$ be the linear transformation given by :

$$T\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} a+b & a+c \\ d+e & d+f \end{bmatrix}.$$

Show that T is onto but not one-to-one.