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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5376**

Unique Paper Code : **2354001201**

Name of the Paper : **Analytic Geometry**

Name of the Course : **Common Prog Group**

Type of the Course : **GE**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts of each question.

Each part carries **7½** marks.

Use of calculator is not allowed.

1. (*a*) The parabola $y^2 = 4x$ is shifted down 2 units and right 3 units to generate the new parabola :

(*i*) Find the equation of the new parabola.

(*ii*) Vertex, focus and directrix of the new parabola.

(*iii*) Plot the new vertex, focus, directrix and sketch the new parabola.

(*b*) (*i*) Decide whether the equation :

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0,$$

represents parabola, hyperbola or ellipse.

P.T.O.

(ii) Find the equation of parabola given axis of parabola is $y = 0$ and passes through (3,1) and (6,2).

(c) The coordinate axes are to be rotated through an angle α to produce an equation for the curve $x^2 + y^2 - xy - 2 = 0$ that has no product term. Find α and the new equation. Identify the curve and plot the same.

2. (a) The axes of an xy-coordinate system are rotated through an angle of $\theta = 45°$ to obtain an $x'y'$-coordinate system. Find an equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinates.

(b) Find the equation of an ellipse, given length of minor axis is 8, foci $(0, \pm 3)$.

(c) Sketch the graph of the hyperbola

$$\frac{y^2}{9} - \frac{x^2}{25} = 1$$

showing its vertices, foci and asymptotes.

3. (a) (i) Find the orthogonal projection of $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ on $\vec{b} = 2\hat{i} + \hat{j}$ and then find the vector component of $\vec{v}$ orthogonal to $\vec{b}$.

(ii) Find $\vec{u} \cdot \vec{v}$ given $\|\vec{u}\| = 1$, $\|\vec{v}\| = 2$ and angle between $\vec{u}$ and $\vec{v}$ is $\frac{\pi}{3}$.

(*b*) (*i*) Find area of triangle with vertices (1, 5, –2), (0, 0, 0) and (3, 4, 1).

(*ii*) Find two unit vectors orthogonal to both $\vec{u} = -4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 2\hat{i} + 4\hat{k}$.

(*c*) Find magnitude and equation of line of shortest distance between the lines :

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

4. (*a*) Find the distance between lines

L1 : $x = 1 + 4t$, $y = 5 - 4t$, $z = -1 + 5t$ and L2 : $x = 2 + 8t$, $y = 4 - 3t$, $z = 5 + t$.

(*b*) Find the parametric equation of a line which passes through P(1, –7, 8) and Q(2, 3, –5) . Also, find its distance from $4x + y + 7z + 9 = 0$.

(*c*) Locate the point of intersection of the plane $2x + y - z = 0$ and the line through (3, 1, 0) that is perpendicular to the plane.

5. (*a*) A point moves so that the sum of the squares of its distances from the six faces of a cube is constant. Show that its locus is a sphere.

(*b*) Find the equation of the circle circumscribing the triangle formed by the three points (a, 0, 0), (0, b, 0) and (0, 0, c).

(*c*) Find the equation of the sphere through the circle

$$x^2 + y^2 + z^2 = 1, \; 2x + 4y + 5z = 6$$

and touching the plane $z = 0$.

6. (*a*) Find the equation of a cone with vertex (5, 4, 3) and $3x^2 + 2y^2 = 6$, $y + z = 0$ as base.

(*b*) Find the equation of the cone whose vertex is at the point (α, β, γ) and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$.

(*c*) Find the equation of a cylinder whose generating lines have the direction cosines (l, m, n) and passes through the circle :

$$x^2 + y^2 = a^2, \; y = 0.$$