700

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5906**

Unique Paper Code : **2353510001**

Name of the Paper : **DSE : Elementary Mathematical Analysis**

Name of the Course : **B.Sc. (Physical Science and Mathematical Science)**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

All questions carry equal marks.

The use of the calculator is not allowed.

1. (*a*) Use the sequential criterion for limits of functions to prove that :

$$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$$

does not exist. Also, check for the continuity of the function :

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases},$$

at $x = 0$. 4,3.5

P.T.O.

(*b*) Show that the Dirichlet function :

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

is nowhere continuous. 7.5

(*c*) Write the statement of the Intermediate Value Theorem and find the solution of the equation $x^3 - 2x^2 + 4x - 1 = 0$ correct upto 2 decimal places. 2.5,5

2. (*a*) Determine whether the function $f(x) = 2x^2 - 5x + 2$ is uniformly continuous on [–3, 1]. Also, check it on (–2, 2). 4,3.5

(*b*) Write the statement of the fixed-point theorem and prove it. 2.5,5

(*c*) Suppose I is an interval and $f : I \to R$ is continuous, then show that $f(I)$ is an interval. 7.5

3. (*a*) Find the upper and lower Darboux's integrals for $f(x) = x^2 + 3x$, $x \in [1, 2]$. Show that f is integrable on the closed interval [1, 2] and find $\int_1^2 f(x)\,dx$. 4,2,1.5

(*b*) Prove that every continuous function is integrable on [*a*, *b*]. Is the converse true ? Justify your answer. 5,2.5

(*c*) Show that the function :

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$$

is not integrable on any closed interval [*a*, *b*]. 7.5

4. (*a*) If f and g are integrable over $[a, b]$, then, prove that $f - g$ is also integrable over $[a, b]$. 7.5

(*b*) Find functions $f, g : [0, 1] \to \mathbf{R}$ such that f and $f \circ g$ are integrable on $[0, 1]$, but g is not. 7.5

(*c*) State the Fundamental Theorem of Calculus (First Form). Use it to calculate :

$$\int_0^{\pi} \sin(x)\,dx.$$ 2.5,5

5. (*a*) Show that the sequence $\langle f_n \rangle$, where :

$$f_n(x) = nx\, e^{-nx^2},\ n \in \mathbf{N},$$

converges pointwise to zero on $[0, 1]$, but not uniformly convergent on $[0, 1]$. 3.5,4

(*b*) Show that the series :

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + n^4x^2},\ x \in \mathrm{R}$$

is uniformly convergent. 7.5

(*c*) Find the radius of convergence of the power series :

$$\sum_{n=1}^{\infty} \frac{2n!}{n!} x^{2n}.$$ 7.5

P.T.O.

6. (*a*) Show that the series :

$$\sum \frac{(-1)^{n+1}}{n}|x|^n$$

is uniformly convergent in [–1, 1]. 7.5

(*b*) Find the radius of convergence of the power series :

$$\sum_{n=1}^{\infty} \frac{z^n}{n(\log n)^2}.$$ 7.5

(*c*) Give an example to show that a series is convergent but is not uniformly convergent. Prove it with the help of an example. 7.5