(c) Show that, as the parameter c decreases, the positive equilibrium point of the price model :

$$P_{n+1} = c + P_n + \frac{1}{P_n}$$

undergoes a bifurcation.

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[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7602** **I**

Unique Paper Code : 2353200010

Name of the Paper : Discrete Dynamical Systems

Name of the Course : **B.A./B.Sc. Programme - DSE**

Semester : VI

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) Each question contains **three** Parts. Attempt **all** questions by selecting any **two** Parts from each question.

(iii) Parts of questions to be attempted together.

(iv) **All** questions carry equal marks.

(v) All parts from each question carry 7.5 Marks.

(vi) Use of non-programmable scientific calculator is allowed.

1. (a) Construct the linear prey-predator model with no immigration, migration or harvesting and :

(i) The prey's intrinsic growth rate is 1.3 and the predator's is 1.4. The prey population is reduced by 0.3 times the predator population, while the predator's population increases by 0.2 times the prey population.

(ii) If both populations were isolated, they would remain constant. However, when interacting, the prey is reduced by 1.5 times the predator population, and the predator poplation is augmented by 1.5 times the prey population.

(b) Construct the linear competition model that satisfies :

(i) The intrinsic growth rate of the first species is 1.6 and that of the second is 1.3; each species' growth is reduced by 0.35 times the population of the other species.

(c) Find all non-negative fixed points of the given nonlinear models and use the derivative to determine their stability

$$P_{n+1} = rP_n\left(1 - \frac{P_n}{1000}\right) \text{for } (a)\, r = 0.5; (b)\, r = 2$$

6. (a) Define superstable fixed point. Find the parameter value for which the positive fixed point of

$$f_r(x) = rx(1-x)$$

(b) The equation $P_{n+1} = r\,P_n\left(1 - \frac{P_n}{C}\right) - k$ may be viewed as a density dependent population model with constant mitigation or harvesting. For r = 4, C = 800 and k = 200 this model has two equilibrium points and a 2-cycle.

(i) Find the two equilibrium points and determine their stability

(ii) Find the points of the 2-cycles and determine the stability of that cycle.

(b) For the prey-predator model with constant immigration $P_{n+1} = 0.8P_n - 0.3Q_n + 8000;$ $Q_{n+1} = 0.2P_n + 0.9Q_n + 2000$. Find the fixed point and determine whether it is a sink, source or saddle why iterating and graphing in solution space the first few iterates for several choices of initial conditions.

(c) Find the equilibrium population levels, if any, for Nonlinear Population model $P_{n+1} = 2P_n e^{-P_n/10000}$. Also discuss their stability.

5. (a) In each of the following identify the function f (x) begin iterated :

(i) $x_{n+1} = 2x_n - \dfrac{7}{x_n}$

(ii) $x_{n+1} = \dfrac{5x_n}{x_n + 8}$

(b) Compute x_1, x_2, x_3, x_4, x_5, if (a) $x_{n+1} = 1/x_n$ and $x_0 = 2$; (b) $x_{n+1} = 5/x_n$ and $x_0 = 8$. (c) From these can you predict what happens to all solutions of $x_{n+1} = \dfrac{c}{x_n}$?

(ii) In addition, the first species receives a constant immigration of 3000 individuals per time step, while the second species experiences emigration of 1500 individuals per time step.

(c) Construct the linear competition model with no immigration, migration or harvesting that satisfies :

(i) $r_1 = 2.5, r_2 = 4/3, P_0 = 1200,$
$Q_0 = 1400, P_1 = 1300, Q_1 = 850$

(ii) $P_0 = 150, Q_0 = 250, P_1 = 180,$
$Q_1 = 120, P_2 = 300, Q_2 = 50$

2. (a) Construct the linear overlapping-generations model with no immigration, migration or harvesting, that satisfies :

(i) $r = 1/2, P_{-1} = 4500, P_0 = 6500, P_1 = 6500$

(ii) $P_{-1} = 100, P_0 = 450, P_1 = 600, P_2 = 2500$

(b) The following are overlapping-generations models that have been converted into systems. Find the original equation for P_n :

(i) $P_{n+1} = 1.6P_n + 0.4Q_n, Q_{n+1} = P_n$

(ii) $P_{n+1} = 0.9P_n + Q_n, Q_{n+1} = P_n + 500$

(c) For the price-demand model

$$\begin{cases} P_{n+1} = P_n + D_n + h_1, \\ D_{n+1} = -2P_n + D_n + k_1, \end{cases}$$

answer the following :

(i) When $D_0 = 80$, what is the minimum value of h_1 that will ensure $P_1 \geq P_0$?

(ii) When $P_0 = 30$, what is the maximum value of k_1 that will ensure $D_1 \leq D_0$?

3. (a) Suppose someone borrows \$1000 at an interest rate of 12% per year paid monthly. Give the iterative equation that models the unpaid balance on the loan if :

(i) \$50 is paid back at the end of each month;

(ii) the borrower pays back nothing at the end of the first month, \$10 at end of the second month, \$20 at the end of the third month, \$30 at the end of the fourth month, etc.

(iii) Create a time series graph for the first 10 data points (n, P_n).

(b) Suppose that a savings account pays 4% annual interest compounded monthly. If \$6000 is initially deposited :

(i) How much will be in the account after 4 years ?

(ii) When will the account balance reach \$8000 ?

(c) Suppose that at each month \$200 is deposited into a savings account with an original balance of \$1500 that pays 4% annual interest compounded monthly.

(i) What will the account balance be 2 years later ?

(ii) How much interest will the account earn during that time ?

4. (a) Suppose that in a certain part of the country 90% of the children of college educated parents get a college education, but 10% do not. Also among parents who are not college educated, 40% of their children get a college education, but the other 60% do not. Assuming that the current distribution of parents in that region includes 35% who are college educated and 65% who are not, what will that distribution be three generations from now ?