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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8085**

Unique Paper Code : **2354001202**

Name of the Paper : **Introduction to Linear Algebra**

Name of the Course : **Common Prog Group**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

1. (*a*) Let x and y be vectors in $\mathbf{R}^n$. Prove that $|x \cdot y| \leq \|x\| \, \|y\|$.

(*b*) Solve the following system of linear equations using Gauss-Elimination method :

$$3x_1 + x_2 + 7x_3 + 2x_4 = 13$$

$$2x_1 - 4x_2 + 14x_3 - x_4 = -10$$

$$5x_1 + 11x_2 - 7x_3 + 8x_4 = 59.$$

(*c*) Using Gauss-Jordon method, find the complete solution set for the following system of equations :

$$2x_1 + x_2 + 3x_3 = 16$$

$$3x_1 + 2x_2 + x_4 = 16$$

$$2x_1 + 12x_3 - 5x_4 = 5.$$

P.T.O.

2. (*a*) Define Rank of a matrix. Find the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}.$$

(*b*) Find the characteristic polynomial of the matrix :

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & -4 \end{bmatrix}.$$

Also, find all the eigenvalues and eigenspaces for A.

(*c*) Is the matrix :

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

diagonalizable ? Justify.

3. (*a*) (*i*) Show that the set of all singular 2×2 matrices under the matrix addition and scalar multiplication is not a vector space.

(*ii*) Show that the set of all non-singular 2×2 matrices under matrix addition and scalar multiplication is not a vector space.

(*b*) Use Simplified Span Method to find a simplified general form for all the vectors in Span(S), where :

$$S = \left\{ \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} -2 & -5 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ -3 & 4 \end{bmatrix} \right\}$$

is a subset of M_{22}.

(c) Show that the subset {[1, 1, 1, 1], [1, 1, 1, –1], [1, 1, –1, –1], [1, –1, –1, –1]} of $\mathbf{R}^4$ forms a basis of $\mathbf{R}^4$.

4. (a) Show that the set of vectors of the form [$2a - 3b$, $a - 5c$, a, $4c - b$, c] in $\mathbf{R}^5$ forms a subspace of $\mathbf{R}^5$ under the usual operations.

(b) Use Independent Test Method to find whether the set S = {[–2, 4, 2], [–1, 5, 2], [3, 5, 1]} of vectors is linearly independent or linearly dependent.

(c) Determine whether the vector X = [2, 2, –3] is in the row space of the matrix :

$$A = \begin{bmatrix} 4 & -1 & 2 \\ -2 & 3 & 5 \\ 6 & 1 & 9 \end{bmatrix}.$$

If so, then express [2, 2, –3] as a linear combination of the rows of A.

5. (a) Define the linear transformation. Determine whether the following function is a linear transformation :

$T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ given by $T([x_1, x_2, x_3]) = [x_1 + 1, x_2 - 2, x_3]$.

(b) Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the linear operator that performs a counterclockwise through an angle 60°. Find the matrix representation for T with respect to the ordered basis B : {[1, 2], [3, 4]} for $\mathbf{R}^2$.

(c) Let $T : P_3(\mathbf{R}) \to \mathbf{R}^3$ be given by :

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = [a_0 + a_1, 2a_2, a_3 - a_0].$$

Find the matrix for T with respect to standard bases for $P_3(\mathbf{R})$ and $\mathbf{R}^3$. Use this matrix to calculate $T(5x^3 - x^2 + 3x + 2)$ by matrix multiplication.

6. (a) Let $T : \mathbf{R}^4 \to \mathbf{R}^3$ be the linear transformation given by $T(X) = AX$, where :

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}.$$

(i) Is $[-7, 3, 0, 1]$ in Ker(T) ? Justify.

(ii) Is $[2, 3, 8]$ in Range(T) ? Justify.

(b) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear operator given by :

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 5 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find a basis for Ker(T) and a basis for Range(T). Also, verify that :

$$\dim(\text{Ker}(T)) + \dim(\text{Range}(T)) = \dim(\mathbf{R}^3).$$

(c) Let $T : M_{22}(\mathbf{R}) + M_{23}(\mathbf{R})$ be the linear transformation given by :

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a - b & 0 & c - d \\ c + d & a + b & 0 \end{bmatrix}.$$

Show that T is one-to-one but not onto.