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Your Roll No.....

Sr. No. of Question Paper : 8641

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Unique Paper Code : 2272101202

Name of the Paper : Intermediate Mathematical Methods for Economics

Name of the Course : ITEP

Semester : II, 2025

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. The use of a simple calculator is allowed.
3. Attempt any **nine** questions.
4. **All** questions carry equal marks.
5. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

- Q1. (a) Find the equation of a plane through (1, -1, 3) parallel to the plane $3x + y + z = 7$ (5)
- (b) Find the equation of the plane passing through the points (3, 4, -3), (5, 2, 1) and (2, -1, 4). (5)

- Q2. (a) A firm has a fixed labour force and two plants that each produce three goods. The output of the first plant is $x = (8, 4, 4)$ and that of the second plant $y = (2, 6, 10)$, when the entire labour force is assigned in any manner. If a fraction λ ($0 \leq \lambda \leq 1$) is allocated to the first plant, its output is λx , and similarly for the second plant. (a) Is it possible for the firm to produce either of the output vectors (5, 5, 7) and (7, 4, 5) if the outputs cannot be thrown away? How does your answer change if output can be thrown away? (5)

- (b) Solve the following equation by using Cramer's Rule (5)

$$2x + y + z = 7,$$

$$3x - y - z = -2 \text{ and}$$

$$x + 2y - 3z = -4$$

- Q3. (a) The prices, in rupees per unit, of the three commodities X, Y and Z are x , y and z respectively. A purchases 4 units of Z and sells 3 units of X and 5 units of Y. B purchases 3 units of Y and sells 2 units of X and 1 unit of Z. C purchases 1 unit of X and sells 4 units of Y and 6 units of Z. In the process A, B and C earn Rs 6000, 5000 and 13000 respectively. Using matrices, find the prices of three commodities (note that selling the unit is positive earning and buying the units is negative earning.) (6)

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- (b) Let D be the 3×3 diagonal matrix with entries d_1, d_2 and d_3 along the diagonal and zeros elsewhere. Let $a = (a_{ij})$ be an arbitrary 3×3 matrix. Compute AD and DA . Show that AD multiplies the i th column of A by entry d_i while DA multiplies the i th row of A by entry d_i (4)
- Q4. In a Leontief system (10)
- (a) What is the interpretation of the condition that $a_{ii} = 0$ for all i
- (b) What is the interpretation of sum $a_{i1} + a_{i2} + \dots + a_{in}$
- (c) What is the interpretation of the input coefficient $(a_{1j}, a_{2j}, \dots, a_{nj})$?
- (d) Can you give the interpretation to the sum $a_{1j} + a_{2j} + \dots + a_{nj}$
- Q5. (a) Given the demand function: $p_1 = 180 - 15x_1$ and $p_2 = 380 - 20x_2$ in two markets of discriminating monopolist and his total cost function is $C = 150 + 120(x_1 + x_2)$, find the optimal quantities and prices for maximum profit? (5)
- (b) Find the maximum and minimum values of $U = 2x^2 - 2xy + y^2 + 5x - 3y$ (5)
- Q6. (a) A consumer's demand for a good A is given by $Q_A = 4850 - 5P_A + 0.1Y + 1.5P_B$ where P_A and P_B are the prices of goods A and B and Y is the income of the consumer. Find the partial elasticity of demand for A with respect to P_B when $P_A = 200$, $P_B = 100$, and $Y = 10000$. Are goods A and B complements or substitutes? (5)
- (b) When is the vector b said to be a linear combination of vectors x, y and z . Consider the vectors $u = (1, 2, 3)$ and $v = (2, 3, 1)$. Find k so that $w = (1, k, 4)$ is a linear combination of u and v . (5)
- Q7. (a) Find the domain of the function given by the following formula and sketch this set
- $$f(x, y) = \frac{2}{\sqrt{(x^2 + y^2 - 4)}} + \sqrt{9 - (x^2 + y^2)} \quad (5)$$
- (b) Find all the first and second order partials of the function
- $$w(x, y, z) = 3xyz + x^2y - xz^3 \quad (5)$$
- Q8. (a) Compute $\frac{dz}{dt}$ when $z = f(x, y) = x^2 + y^2$ with $x = t^2$ and $y = 2t$ (5)
- (b) Show that elasticity of substitution is always unity in case of Cobb Douglas Production function. (5)
- Q9. (a) Compute the directional derivatives of the following functions at the given point and in the given direction: $f(x, y) = 2x + y - 1$ at $(2, 1)$ in the direction of $(1, 1)$ (5)
- (b) Determine the definiteness of the following quadratic form $4x^2 + 8xy + 5y^2$ (5)
- Q10. Examine concavity/convexity of the following functions:
- (a) $f(x, y) = 2xy - x^2 - 2y^2 - 3x + 4$ (5)
- (b) $f(x, y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$ (5)