

This question paper contains 4 printed pages]

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S. No. of Question Paper : 5673

Unique Paper Code : 2352012403

Name of the Paper : Numerical Analysis

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any two parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. (a) Define order of convergence of a sequence $\{p_n\}$ which converges to a number p . The sequence generated by the formula $x_{n+1} = \frac{x_n^3 + 3x_n a}{3x_n^2 + a}$ converges to $\sqrt{a}$. Determine the order of convergence and the asymptotic error constant.
- (b) Perform four iterations of Bisection method to approximate the root of the equation $x^4 - x - 10 = 0$, in the interval $(1, 2)$.
- (c) Explain geometrically the Newton's method to determine a root p of an equation $f(x) = 0$. Hence, using initial approximation $p_0 = 2$, perform two iterations to obtain approximation to the value of $\sqrt[3]{17}$.

P.T.O.

2. (a) Use Newton's method to approximate the root of the equation $\ln(1 + x) - \cos(x) = 0$, in the interval $(0, 1)$. Use error tolerance $= 10^{-4}$, and initial approximation $p_0 = 0.5$.
- (b) Define a fixed point of a function. Let g be a continuous function on the closed interval $[a, b]$, with $g : [a, b] \rightarrow [a, b]$. Furthermore, suppose that g' is continuous on the open interval (a, b) , and there exists a positive constant k such that $|g'(x)| \leq k < 1$, $\forall x \in (a, b)$. Prove that if $g'(p) \neq 0$, then for any $p_0 \in [a, b]$, the sequence $\{p_n\}$, where $p_n = g(p_{n-1})$, converges only linearly to the fixed point p .
- (c) Perform four iterations of Secant method to approximate the root of $x^4 - 7x^2 + 3 = 0$, in the interval $(0, 1)$, using initial approximations $p_0 = 0$ and $p_1 = 1$.
3. (a) For the following system of equations :

$$8x + 5y + 2z = 8,$$

$$6x + 9y + 2z = 11,$$

$$4x + 3y + 8z = 1,$$

starting with the initial approximation as $(0, 0, 0)$, using Gauss Jacobi method, find approximate solution by performing three iterations.

- (b) For the following system of equations :

$$20x + y - 2z = 17,$$

$$3x + 20y - z = -18,$$

$$2x - 3y - 20z = 25,$$

starting with initial approximation as $(0, 0, 0)$, using Gauss Seidel method, find the approximate solution by performing three iterations.

- (c) Find the LU decomposition by taking $u_{ii} = 1$ for the matrix :

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}$$

4. (a) Construct the Newton form of interpolating polynomial for a function f passing through the points $(0, 1)$, $(1, 3)$ and $(3, 55)$. Hence, evaluate $f(2.5)$.
- (b) Let $f(x) = \ln(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use Lagrange interpolation to calculate an approximate value for $f(1.04)$ and obtain a bound on the truncation error at $x = 1.04$.
- (c) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the data :

x	$f(x)$
1	3
2	7
4	21
8	73

5. (a) Derive the central difference approximation to the second order derivative of a function f at a point $x = x_0$

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

- (b) Verify that the following difference approximation

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}$$

for the first order derivative provides the exact value of the derivative, regardless of the values of h , for the functions $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$, but not for the function $f(x) = x^3$.

- (c) By using Trapezoidal rule, approximate the value of the integral $\int_2^3 (x + \ln x) dx$. Further, calculate the absolute error, theoretical error bound and compare them.

6. (a) Define the degree of precision of a quadrature rule. Further, calculate the degree of precision of Simpson's rule.
- (b) Use Euler's method to determine the approximate solution of the initial value problem $y'(x) = xy^3 - y$, $0 \leq x \leq 2$ such that $y(0) = 2$, by taking the step size as $h = 1/2$.
- (c) Use Modified Euler's method to determine the approximate solution of the initial value problem : $y'(x) = 3x - y/x$, $1 \leq x \leq 2$, such that $y(1) = 2$, by taking the step size as $h = 1/2$.