(c) State the relationship between the orbital angular momentum quantum number l and the magnitude of angular momentum. If an electron is in a 3p orbital, what are the values of n, l, and possible ml. (7+7+4)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4820 **J**

Unique Paper Code : 2224002006

Name of the Paper : Quantum Mechanics

Name of the Course : **B.Sc. Hons. - (Physics)_NEP: UGCF-2022**

Semester : IV (GE Paper)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions in total.
3. Question No. **1** is compulsory.

1. Attempt any **six** of the following : (6×3=18)

   (a) State linearity and superposition principle.

P.T.O.

(b) If a particle is localized within a region of width $\Delta x = 1 \times 10^{-10}$ m, calculate the minimum uncertainty in its momentum. (Take h = 1.05×10^{-34} Js).

(c) Evaluate the commutator [x, p].

(d) How does the uncertainty in position and momentum evolve over time for a free Gaussian wave packet?

(e) Explain how the Heisenberg uncertainty principle leads to the concept of zero-point energy.

(f) A particle in a one-dimensional infinite potential well of width a has a wave function in the region $0 < x < a$ given by :

$$\psi(x) = A\sin\frac{\pi x}{a}$$

and $\psi(x) = 0$ outside the well. Find the normalization constant A.

(b) Using the commutation relations and properties of ladder operators, derive the energy eigenvalues of the harmonic oscillator. Explain how the ladder operators can be used to construct the eigenstates starting from the ground state. (10+8)

6. Consider the hydrogen atom, where the potential energy of the electron in the electrostatic field given by :

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r}$$

(a) Starting from the time-independent Schrödinger equation in spherical coordinates, perform the separation of variables and derive the differential equations for the radial and angular parts of the wave function.

(b) Solve the angular part of the equation and show that the solutions are spherical harmonics $Y_m^l(\theta, \varphi)$. Identify the quantum numbers that arise from the angular equations and explain their physical significance. State the allowed values of the quantum numbers l and m.

(ii) Repeat the derivation for the case $E < V_0$ and interpret the physical meaning of the solution. Why is the transmission coefficient non-zero even when the particle classically cannot pass the potential step? (8+10)

5. A particle of mass m is confined in a one-dimensional quantum harmonic oscillator potential

$$V(x) = \frac{1}{2} m\omega^2 x^2$$

(a) Using the algebraic method (ladder operator technique), define the annihilation operator a and creation operator $a^\dagger$ in terms of the position x and momentum p operators. Show that the Hamiltonian of the harmonic oscillator can be written as:

$$H = \hbar\omega(a^\dagger a + \tfrac{1}{2})$$

(g) Consider a particle in one dimension with the normalized wave function:

$$\psi(x) = A\, e^{ikx}$$

where A is a constant and k is a real number. Calculate the expectation value of momentum.

2. (a) State the conditions that the wave function $\psi(x)$ and its derivative must satisfy at the boundaries between two regions with different potentials in the one-dimensional Schrödinger equation. Explain the physical reasons behind these continuity requirements and their implications about the behaviour of a particle encountering a sudden change in potential?

(b) How do these boundary conditions lead to the quantization of energy levels in bound systems?

(c) Starting from the time-dependent Schrödinger equation, derive Ehrenfest's theorem for a one-dimensional particle moving in a potential V(x).

(6+6+6)

3. Consider a particle of mass m in a one-dimensional finite square potential well defined as :

$$V(x) = 0 \quad |x| < a$$

$$= V_0 \quad |x| > a$$

where V_0 is the potential barrier and a is the half-width of the well.

(a) Write down the time-independent Schrödinger equation for regions inside the well and outside the well.

(b) Solve the Schrödinger equation in each region and apply the boundary conditions for $\psi(x)$ and its derivative at $x = \pm a$ to obtain the equations governing the allowed energy levels.

(c) Discuss qualitatively how the number of bound states depends on the depth V_0 and the width 2a of the well.

(d) Sketch the wave functions for the lowest two bound states and explain how their shapes are influenced by the finite height of the potential well.

(3+5+5+5)

4. A particle of mass m and total energy E is incident from the left on a one-dimensional potential of the form :

$$V(x) = V_0 \quad x \geq 0,$$

$$= 0 \quad x < 0$$

(i) Solve the time-independent Schrödinger equation in both regions and derive the expressions for the reflection coefficient R and transmission coefficient T for the case $E > V_0$.