

- (b) The length of the skulls of 10 fossil skeletons of an extinct species of bird has a mean of 5.68 cm and a standard deviation of 0.29 cm. Assuming that such measurements are normally distributed, find a 95% confidence interval for the mean length of the skulls of this species of bird. (Given that $Z_{0.025} = 1.96$, $t_{0.025,9} = 2.262$.) (9,6)

7. (a) What is a statistical hypothesis? Define the terms: (i) level of significance, (ii) best critical region, and (iii) null and alternative hypotheses.

- (b) Given the frequency function:

$$f(x, \theta) = \begin{cases} \frac{1}{\theta}, & 0 \leq x \leq \theta \\ 0, & \text{elsewhere} \end{cases}$$

and that you are testing the hypothesis $H_0: \theta = 1$, against $H_1: \theta = 2$ by means of a single observed value of x . What would be the sizes of the type I and type II errors, if the interval $x \geq 0.5$ is the critical region? Also obtain power of the test.

(7, 8)

8. Write short notes on any three:

(i) Two types of errors

(ii) UMP test

(iii) UMPU test

(iv) Method of least square (15)

(500)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4868

J

Unique Paper Code : 2374002004

Name of the Paper : Basics of Statistical Inference

Name of the Course : **Statistics: Generic Elective under NEP-UGCF**

Semester : IV

Duration : 3 Hours — — — Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt **Six** questions in all selecting Three from each section.
3. Attempt **all** parts of question in continuation.
4. Use of simple calculator is allowed.

Section I

1. (a) What do you understand by point estimation? When would say that estimator of a parameter is good?

P.T.O.

In particular, discuss the requirements of consistency and efficiency of an estimator with illustrations.

- (b) Show that $s^2 = \frac{1}{n} \sum (X_i - \bar{X})^2$ is a biased estimator of σ^2 , where $\bar{X}$ is the mean of the random sample $X_1, X_2, \dots, X_n$. Also, show that $S^2 = \frac{n}{n-1} s^2$ is an unbiased estimator of σ^2 .

(9,6)

2. Define Minimum variance unbiased estimator (MVUE) and Minimum variance bound estimator (MVB) and explain clearly the difference between them. Given a random sample of size n from a population with p.d.f.:

$$f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}; \quad x > 0, \quad \theta > 0$$

Show that the mean is the MVB estimator of the parameter θ . (15)

3. (a) Discuss the terms:
- (i) estimate,
 - (ii) sufficient statistic, and
 - (iii) completeness of a statistic.

- (b) Explain Fisher-Neyman criterion. A random sample $X_1, X_2, \dots, X_n$ is drawn from a Normal population $N(\mu, \sigma^2)$. Find sufficient estimators for μ and σ^2 . (6,9)

- 4: Write short notes on any three:

- (i) Unbiased estimator
- (ii) Cramer-Rao inequality
- (iii) Rao-Blackwell Theorem
- (iv) Lehmann-Scheffé Theorem (15)

Section II

5. (a) In random sampling from a Normal population $N(\mu, \sigma^2)$, find the MLES for
- (i) μ , when σ^2 is known,
 - (ii) σ^2 , when μ is known, and
 - (iii) the simultaneous estimation of μ and σ^2 .
- (b) Show by means of an example, that MLEs are not necessarily unbiased. (9, 6)
6. (a) Explain the method of constructing $100(1-\alpha)\%$ confidence interval for μ of Normal population with mean μ and variance σ^2 . How do you proceed if σ^2 is (i) known, and (ii) unknown.