(c) Given the following data.

x	2	3	4
y	5	7	9

Set up the residuals to fit the model $y = cx$ to the data set Using the sum of Absolute Deviations Criterion (not solve for c).

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5107 **J**

Unique Paper Code : 2354000010

Name of the Paper : Introduction to Mathematical Modelling (GE)

Name of the Course : **B.Sc.(H)/B.A.(H)/B.Com.(H)/BMS**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt **any two** parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable Scientific Calculator is allowed.

1. (a) Solve the system of differential equations:

$$\frac{dx}{dt} = -k_1 x,\ x(0) = x_0$$
$$\frac{dx}{dt} = k_1 x - k_2 y,\ y(0) = 0,\ k_1 \neq k_2$$

Here k_1 & k_2 describe rates at which the drugs move between the two sequential compartments (the GI-tract and the bloodstream) and I denotes the amount of drug released into the GI-tract in each time step. Here $x(t)$ is the level of drug in the GI- tract and $y(t)$ is the level in the bloodstream at time t.

(b) Consider two American Lakes where Lake Erie flows into the Lake Ontario. Lake Erie has volume $V = 458\times10^9$ cube meter and flow rate of water into the lake $F = 480\times10^6$ cube meter per day.

(b) Fit the curve $y = ax^2$ by using Least Square Method where

x	5	10	15	20	25
y	2	4	7	12	20

(c) Use the Middle Square Methods to generate 10 random numbers using

x_0 = 7182 (seed no).

6. (a) For the following data formulate the mathematical model that minimizes the largest deviation between the data and model $y = ax^2 + bx + c$.

x	0.1	0.2	0.3	0.4	0.5
y	0.06	0.12	0.36	0.65	0.95

(b) Generate random seven Numbers using Linear Congruence Method $x_i \equiv (ax_{i-1}+b) mod\ (m)$ where m = 123, a= 5 and b = 2,seed no x_0 = 73

P.T.O.

4. (a) Develop a model with three differential equations describing a predator-prey interaction, where there are two different species of prey and one species of predator.

(b) Construct an appropriate input-output compartment model and associated word equations for the number of soldiers in both the red and blue armies.

(c) Determine the Predator and prey model in limiting cases of prey with no predator and predator with no prey.

5. (a) On flipping a die the probability of each event occurrence is 1/6 Using

Monte Carlo Simulation write Monte Carlo Roll of a Fair Die Algorithm

Lake Ontario has volume $V = 1636\times10^9$ cube meter and flow rate of water into the lake $F=572\times10^6$ cube meter per day. How long will it take for the lake's population level to reach 5% of its initial level, if only fresh water flows into the lake.

(c) Write the balance law and find the differential equation for the lake pollution model. And also find the solution of that differential equation.

2. (a) The earth's atmosphere P is often modelled by assuming that $\frac{dP}{dh}$ (the rate at which dh pressure P change with altitude h above sea level) is proportional to P.Suppose that the pressure at sea level is 1013 millibars and that the pressure at an altitude 20 km is millibars.

(i) Solve the differential equation $\frac{dP}{dh} = -kP$. Determine k and the constant of integration from the initial conditions.

(ii) ii. What is the atmosphere pressure at an altitude of 50 km?

(b) Solve the logistic differential equation.

(c) Let N(t) be the number of ranchers who have adopted an improve pasture technology in Uruguay. Then N(t) satisfies the differential equation: $\frac{dN}{dt} = aN\left(1 - \frac{N}{N^*}\right)$ where N* is the total population of ranchers. According to Banks (1994), N* = 17015, a = 0.490 & N_0 = 141. Determine how long it takes for the improved pasture technology to spread to 80% of the population.

3. (a) Create a compartmental diagram for the model and formulate suitable word equations for the rates of change in susceptible individuals and contagious infections.

(b) Develop a model with three differential equations describing a predator-prey interaction, where there are two different species of prey and one species of predator.

(c) Imagine a disease in which every individual infected stays contagious indefinitely (they do not recover). Disregard any instances of birth and death. Create a set of differential equations representing the quantity of susceptible individuals S and the number of currently infected individuals I.