$$f_a(x) = x\sqrt{a - x}.$$

(c) Determine whether the positive fixed point

$$p_a = \sqrt{a-1} \text{ of } f_a(x) = x(a - x^2),$$

bifurcates for some $a > 1$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5108 **J**

Unique Paper Code : 2354000011

Name of the Paper : Discrete Dynamical Systems

Name of the Course : **Common Programme Group-GE**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Each question contains **three parts.**
3. Attempt **all** questions by selecting **any parts from each question.**
4. **Non-Programmable scientific calculators** are allowed.
5. Parts of questions to be attempted **together.**
6. **All questions** carry equal marks.

1. (a) Construct the linear prey-predator model with no immigration, migration or harvesting, and:

    (i) The prey grows at an intrinsic rate of 1.1 and the predator at 1.2.The prey's growth is decreased by 0.25 times the number of predators, whereas the predator's increase is supported by 0.35 times the prey population.

    (ii) In the absence of interaction, the populations remain stable; however, when interacting the prey population is diminished by 0.8 times the number of predators, and the predator population is boosted by 0.8 times the prey count.

   (b) Construct the linear competition model that satisfies:

    (i) The first species has a growth rate of 1.4 and the second a growth rate of 1.2; both species are inhibited by an interaction term equal to 0.5 times the population of the other

5. (a) Find all fixed points and determine their stability using the formula for the exact solution. Also, obtain the basins of attraction if possible:

$$x_{n+1} = 5\sqrt{x_n}.$$

   (b) Discuss various nonlinear Contagious disease models.

   (c) If $f(x) = \frac{x}{x+1}$.

   Compute and simplify $f^2(x)$ and $f^3(x)$. Find a general formula for $f^n(x)$. Also, show that $f(x)$ has no cycles of period m > 1.

6. (a) Define superstable fixed point. Find the parameter value for which the positive fixed point of

$$f_r(x) = r\,x\,(1 - x).$$

   (b) Define interval of existence and interval of stability for a point. Find the 2-cycles, their intervals of existence and stability for $a > 0$, for $f_a\,(x)$ given by

4. (a) Suppose a population has a growth rate of 1.2 per generation, but is being harvested at a rate of 100 per generation. If its present size is 700, how long will it take before the population exceeds 1400?

(b) If \$200 is deposited every 2 weeks into an account paying 6.5% annual interest compounded bi-weekly with an initial zero balance:

(i) How long will it take before \$10000 is in this account?

(ii) During this time how much is deposited and how much comes from interest?

(c) Suppose that of 1000 businesses being followed, 90% of those that make a profit in any year will also make a profit the following year and 10% will not, and 50% of those that did not make a profit in some year will make a profit in the following year and 50% will not. If presently 750 of those businesses are making a profit and 250 are not, how many will be and how many would not be making a profit in 5 years?

(ii) Moreover, incorporate a constant influx of 2000 individuals per step for the first species and a simultaneous migration (loss) of 1000 individuals per step for the second species.

(c) Determine the linear competition model with no immigration, migration, or harvesting that meets the following initial conditions:

(i) Let the intrinsic growth rates be $r_1 = 1.8$ and $r_2 = \frac{5}{3}$. With initial populations

$P_0 = 1100$ and $Q_0 = 1500$, and after one time step the populations evolve to $P_1 = 1250$ and $Q_1 = 900$, use these data to formulate the model.

(ii) In a second scenario, assume $P_0 = 80$ and $Q_0 = 180$, followed by $P_1 = 120$ and $Q_1 = 90$, and then $P_2 = 200$ and $Q_2 = 40$. Construct the linear model that fits these conditions.

2. (a) The following are overlapping-generations models rewritten as systems. Determine the original equation for P_n:

(i) $P_{n+1} = 2.0\,P_n + 0.2\,Q_n, \quad Q_{n+1} = P_n.$

(ii) $P_{n+1} = 0.8\,P_n + Q_n, \quad Q_{n+1} = P_n + 1500.$

(b) Construct the linear overlapping-generations model with no immigration, migration or harvesting, that satisfies:

(i) $r = 1/2, P_{-1} = 4500, P_0 = 6500, P_1 = 6500.$

(ii) $P_{-1} = 100, P_0 = 450, P_1 = 600, P_2 = 2500.$

(c) Discuss the linear Price-Demand-Supply Model and Price-Demand Model. How they are used in real world economic scenario.

3. (a) Suppose someone borrows $1000 at an interest rate of 12% per year paid monthly. Give the iterative equation that models the unpaid balance on the loan if:

(i) $50 is paid back at the end of each month;

(ii) the borrower pays back nothing at the end of the first month, $10 at end of the second month, $20 at the end of the third month, $30 at the end of the fourth month, etc.

(iii) Create a time series graph for the first 10 data points (n, Pn).

(b) When a population has a birth rate b that its death rate d, then that population will decrease over time and may eventually become extinct. For a population with b=0.1,d=0.3 and initial size of 1000:

(i) Find the equation for the population size P_n after n generations;

(ii) Determine how long it will take for the population to fall below 1, which we interpret as extinction.

(c) Suppose that $10000 is initially deposited into an account paying interest at an annual rate of 8%. How much more will be in the account after 5 years if it gets compound rather than simple interest, if compounding is done:

(i) quarterly?

(ii) monthly?