(b) Verify that Z_5 is an integral domain.

(c) Let f be a ring homomorphism from a ring R to a ring S and let A be a subring of R. If A is an ideal and f is onto S, then show that $f\ (A)$ is an ideal.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 7325 **J**

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A/B.Sc. (Programme) Mathematics as Non-Major/ Minor - DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting any **two** parts from each question.

3. Each part carries **7.5 marks.**

4. Use of Calculator not allowed.

1. (a) State Division Algorithm. For a fixed positive integer $n \neq 1$, if $a \bmod n = a'$ and $b \bmod n = b'$, then prove that $(a + b) \bmod n = (a' + b') \bmod n$ and $(ab) \bmod n = (a'\,b') \bmod n$.

(b) Define an Abelian group. Show that $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} : a \in \mathbb{R}, a \neq 0 \right\}$ is an Abelian group.

(c) Construct Cayley table for $U(20)$. Find the inverse of each element of $U(20)$.

2. (a) State and prove Two-Step Subgroup Test. Use it to prove that if G is an abelian group, then $H = \{a \in G : |a| \text{ finite}\}$ is a subgroup of G, where $|a|$ denote the order of a in G.

(b) State and prove Socks-Shoes property.

(c) Let a be any element of a finite order, say n, in a group G. Show that $\langle a \rangle = \{e, a, a^2, \ldots, a^{n-1}\}$ and $a^i = a^j$ if and only if n divides $i - j$.

3. (a) Find the number of permutations of order 2 in S_4, the group of permutation of degree 4.

(b) Show that $\{I, (13)\}$ is not normal subgroup of S_3, the group of permutation of degree 3.

(c) Show that the factor group $\mathbb{Z}/6\mathbb{Z}$ is a cyclic group of order 6.

4. (a) State Lagrange's theorem for finite group. Show that every group of prime order is cyclic.

(b) Determine all group homomorphism from Z_{12} to Z_{10}.

(c) Show that $\phi : \mathbb{R}^* \to \mathbb{R}^*$ given by $\phi(x) = \frac{x}{|x|}$ is a homomorphism.

5. (a) Define subring and prove that a nonempty set S of a ring R is a subring if it is closed under subtraction and multiplication.

(b) Find all the subrings of a ring Z_{12}.

(c) Define an ideal of a ring and verify that $10\mathbb{Z}$ is an ideal of $\mathbb{Z}$.

6. (a) Define field. Prove that every field is an integral domain.