(iii) Invariance property of a consistent estimator

(iv) Point and Interval estimation

(v) Critical region and Level of Significance

(vi) Difference between MVUE and UMVUE.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 8089 **J**

Unique Paper Code : 2372572401

Name of the Paper : Elements of Statistical Inference

Name of the Course : **B.Sc. (P)/B.A. (P) with Statistics**

Semester : IV (NEP)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt any **Six** questions.

3. **All** questions carry equal marks.

4. Use of non-programable calculator is allowed.

1. (a) Define an unbiased and consistent estimator of a parameter θ in a distribution. Show that $\frac{\sum x_i(\sum x_i - 1)}{n(n-1)}$ is an unbiased estimator of θ^2, for the sample values 1 or 0 with respective probabilities θ and $(1 - \theta)$.

(b) Define Neyman Factorization theorem. Let X_1, X_2, ..., X_n be a random sample from uniform population $[0, \theta]$. Find a sufficient estimator for θ. (8,7)

2. (a) Prove that minimum variance unbiased (MVU) estimator is unique in the sense that if T_1 and T_2 are MVU estimators for $\Upsilon(\theta)$, then $T_1 = T_2$, almost surely.

(b) A random sample (X_1, X_2, X_3) of size 3 is drawn from a normal population with mean μ and variance σ^2. Let T_1, T_2 and T_3 are the estimators of μ, where

(i) If W be a most powerful critical region of size α for testing H_0: $\theta = \theta_0$ against H_0: $\theta = \theta_1$

(ii) If W be uniformly most powerful test of size α for testing H_0: $\theta = \theta$ against H_0: $\theta > \theta_0$.

(8,7)

(b) Let X_1, X_2, ...X_n be a random sample of size n from Normal distribution $N(\theta, \sigma^2)$ where σ^2 is known. Obtain UMP test for testing H_0: $\theta = \theta_0$ against H_0: $\theta > \theta_0$. Also find the power function of the test. sample of size n from Normal distribution (8,7)

8. Explain any five from the followings (5x3=15)

(i) Errors of first and second kinds.

(ii) Rao-Blackwell theorem

P.T.O.

used to determine the most powerful critical region for testing a simple null hypothesis against a simple alternative hypothesis.

(b) Let X have a pdf :

$$f(x,\theta) = \begin{cases} \frac{1}{\theta} e^{\frac{-x}{\theta}} & ; o < x < \infty,\ \theta > 0 \\ 0 ; & elsewhere \end{cases}$$

Find the power of the test for testing $H_0: \theta = 2$ against $H_1: \theta = 1$,, based upon a random sample x1, x2 and critical region: $W = \{((x_1, x_2) : 9.5 \le x_1 + x_2)\}$. (8, 7)

7. (a) Prove that in the critical region W is necessarily unbiased if

$T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 - 4X_2 + 3X_3$ and $T_3 = \frac{\lambda X_1 + X_2 + X_3}{3}$. Then

(i) Find λ such that T_3 is an unbiased estimator for μ.

(ii) Are T_1 and T_2 unbiased estimators of μ?

(iii) Find the best estimator among T_1, T_2 and T_3. (7, 8)

3. (a) Let X be distributed as Poisson (θ). Show that the only unbiased estimator of $e^{\{-(k+1)\theta\}}, k > 0$ is $T = (-K)^x$, where

$$T(x) > 0, if\ x\ is\ even,$$
$$T(x) < 0, if\ x\ is\ odd.$$

(b) State Cramer-Rao inequality with mentioning its underlying regularity conditions. A random sample $x_1, x_2, \ldots\ldots\ldots, x_n$ is taken from a normal population with mean zero and variance σ^2.

Examine if $\sum_{i=1}^{n} x_i^2/n$ is a MVB is a MVB estimator of σ^2.

(7,8)

4. (a) Describe the method of maximum likelihood estimation. Find MLE for the parameter λ of a Poisson (λ) distribution on the basis of a random sample of size n. Also find its variance.

(b) State optimum properties of maximum likelihood estimators. Let $x_2, x_2, \ldots\ldots\ldots x_n$ be a random sample from the uniform distribution with pdf $f(x, \theta) = \begin{cases} 1/\theta, 0 < x < \infty, \theta > 0 \\ 0 \quad , \quad \text{elsewhere} \end{cases}$ Obtain M.L.E. for θ.

(8,7)

5. (a) Obtain $100(1 - \alpha)\%$ confidence limits for parameter $(a)\,\theta$ *and* $(b)\ \sigma^2$ of the distribution

$$f(x;\theta,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2\right\}, -\infty < x < \infty.$$

(b) Let p be the probability that a coin will fall head in a single toss in order to test $H_0: p = \frac{1}{3}$ against $H_1: p = \frac{4}{5}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test.

(8,7)

6. (a) Explain simple and composite hypothesis? Define most powerful test, uniformly most powerful test and unbiased critical region. State the theorem