

6917

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Set up the residuals to fit the model $y = cx$ to the data set. Using the sum of Absolute Deviations Criterion (not solve for c).

(200)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6917

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Unique Paper Code : 2354000010

Name of the Paper : Introduction to Mathematical Modelling (GE)

Name of the Course : **B.Sc.(H)/B.A.(H)/B.Com.(H)/BMS**

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt **any two** parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable Scientific Calculator is allowed.

P.T.O.

1. (a) Define half-life of radioactive isotope. Find relation between half-life (τ) and decay rate (k).
- (b) How long will it take for the lake's pollution level to reach 5% of its initial level, if only fresh water flows into the lake.
- (c) In view of the potentially disastrous effect of overfishing causing a population to become extinct, some governments impose quotas which vary depending on estimates of the population at the current time. One harvesting model that takes this into account is

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{K} \right) - h_0 X$$

- (i) Show that the only non-zero equilibrium population is

6. (a) Using Monte Carlo Simulation find the area trapped between the two curves

$$y = x^2 \text{ and } y = x-6.$$

- (b) Generate five random Numbers using Linear Congruence Method

$$x_i \equiv (ax_{i-1} + b) \bmod (m) \text{ where } m = 123, a = 5 \text{ and } b = 2, \text{ seed no } x_0 = 73$$

- (c) Given the following data.

x	2	3	4
y	5	7	9

x	1	2	3
y	2	5	8

- (b) Fit the curve $y = ax^2$ by using Least Square Method where

x	5	10	15	20	25
y	2	4	7	12	20

- (c) Use the Middle Square Methods to generate 10 random numbers using

$$x_0 = 3043 \text{ (seed no).}$$

$$X_e = K \left(1 - \frac{h}{r} \right)$$

- (ii) At that critical harvesting rate can extinction occur?

2. (a) Solve the initial value problem dt

$$\frac{dN}{dt} = -kN; k > 0, N > 0 \text{ with } N(0) = N_0.$$

- (b) A certain moon rock was found to contain equal numbers of Potassium and Argon atoms. Assume that all the Argon is the result of radioactive decay of Potassium (its half-life is about 1.28×10^9 years) and that one of nine Potassium disintegrates yields an Argon atom. What is the age of rock, measured from the time it only potassium?

(c) Derive the Logistic differential equation.

3. (a) Construct a compartmental diagram for the model and develop appropriate word equation for the rates of change of susceptible and contagious infectives.

(b) Determine a compartmental diagram and appropriate word equation for each of the two populations, the predator and prey.

(c) Consider a disease where all those infected remain contagious for life (they never recover). Ignore any births and deaths. Develop a pair of differential equations for the number of susceptibles S and number of infectives I .

4. (a) Determine the Predator and prey model in limiting cases of prey with no predator and predator with no prey.

(b) Determine the appropriate input-output compartment model and associated word equations for the number of soldiers in both the red and blue armies.

(c) Develop a model with three differential equations describing a predator-prey interaction, where there are two different species of prey and one species of predator.

5. (a) Fit the model $y = cx$ from the following data. Express the three residuals r_1 , r_2 and r_3 corresponding to the data points using Chabyshev Criterion. (Not solve for c)