converse?

(b) Define an alternating series. Test the convergence and absolute convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}.$$

(c) Use the limit comparison test to check the convergence of the series:

$$\sum_{n=1}^{\infty} \frac{\sqrt{5n}-10}{3n+\sqrt{n}}.$$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 8086 **J**

Unique Paper Code : 2354002003

Name of the Paper : Elements of Real Analysis

Name of the Course : **Common Program Group: GE**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting **two** parts from each question.

3. Part of the questions to be attempted together.

4. **All** questions carry equal marks.

1. (a) Define supremum and infimum of a set. Find the supremum and infimum, if exist, of the following sets:

(i) $\left\{1-\frac{1}{n}: n \in \mathbb{N}\right\}$.

(ii) $\{x \in \mathbb{R} : x+2 \geq x^2\}$.

(b) In an ordered field F, prove that:

(i) A set cannot have more than one greatest lower bound.

(ii) If a set contains a greatest lower bound, then that element is the minimum element of that set.

$$\frac{\sqrt{1}}{4\cdot 6}+\frac{\sqrt{3}}{6\cdot 8}+\frac{\sqrt{5}}{8\cdot 10}+\cdots$$

(c) State Root's Test for convergence of a series. Use it to check the convergence of the series

$$\sum_{n=1}^{\infty} 2^{-n-(-1)^n}.$$

6. (a) Let $\sum a_n$ be a series of real numbers. Prove that if $\sum a_n$ an converges absolutely, then the series also converges. What about the

(c) Define Cauchy sequence of real numbers. Further, prove that the sequence

$$\{x_n\} = \{1 + (-1)^n\}$$

is not a Cauchy sequence.

5. (a) Investigate the convergence of the following series and find its sum, if it is

(i) $\sum_{n=0}^{\infty} \frac{3^n+5^n}{4^n}$

(ii) $0.21\overline{75}$

(b) Determine the nth term of the series and check its convergence:

(c) In any ordered field F, define an absolute value of any $x \in F$. Find all $x \in \mathbb{R}$ that satisfy the following inequalities:

(i) $|4x + 5| > 13$

(ii) $\frac{1}{|2x-1|} > 5$

2. (a) In any ordered field F, prove the following properties:

(i) $|x + y| \leq |x| + |y|$.

(ii) If $0 \leq x < y$, then $\frac{x}{1+x} < \frac{y}{1+y}$.

(b) In a complete ordered field F, let A be a nonempty and bounded above subset of F, and let B be the set of all upper bounds of A. Prove that B has a minimum element, and sup A = min B.

(c) Prove that if $c > 0$, then $\lim_{n\to\infty} c^{\frac{1}{n}} = 1$.

3. (a) Define sequence of real numbers. Use definition of convergence of sequence, to show that

$$\lim_{n\to\infty} \frac{5n+2}{n+1} = 5.$$

(b) Let X = $\{x_n\}$ and Y = $\{y_n\}$ be sequences of real numbers that converges to x and y, respectively,

then prove that the sequence X-Y converges to x-y.

(c) Discuss the convergence of the following sequences

(i) $\{x_n\} = \left\{\frac{n}{3n}\right\}$

(ii) $\{x_n\} = \left\{\frac{2^{3n}}{3^{2n}}\right\}$

4. (a) State and prove Monotone Convergence Theorem.

(b) State first squeeze principle for convergence of sequences and use it to prove that

$$\lim_{n\to\infty} \frac{\sin n}{n} = 0.$$