(i) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$.

(ii) $\sum_{n=1}^{\infty} \frac{n^2}{n^3-3}$.



[This question paper contains 8 printed pages.]

**Your Roll No..............**

**Sr. No. of Question Paper : 6929** **J**

Unique Paper Code : 2354002003

Name of the Paper : Elements of Real Analysis

Name of the Course : **Common Program Group: GE**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting **two** parts from each question.

3. Part of the questions to be attempted together.

4. **All** questions carry equal marks.

1. (a) Define bounded and unbounded subsets of $\mathbb{R}$. Give example of each:

   (i) Bounded below but not bounded above.

   (ii) Bounded above but not bounded below.

   (iii) Both bounded above and bounded below.

   (b) Let F be an Archimedean ordered field, $A \subseteq F$, and $u \in F$. Prove that $u = \inf A$ if and only if for all $\varepsilon > 0$,

   (i) for all $x \in A$, $x > u - \varepsilon$, and

   (ii) there exists $x \in A$, such that $x < u + \varepsilon$.



6. (a) State D'Alembert's Ratio Test for the convergence of positive term series. Use it to test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}.$$

   (b) Define absolute and conditional convergence of a series. Test the absolute convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cos n\alpha}{n^3}$$

   $\alpha$ being real.

   (c) Check the convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+4)}{(n+1)\,(n+2)}$$

is convergent.

(b) Prove that a non-negative series converges if and only if its sequence of partial sums is bounded above.

(c) Determine the convergence of the following series:

$$\sum_{k=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdot 8 \cdots (2k)}.$$



(c) State ordered axioms of a field. Prove that the set $\mathbb{N}$ of natural numbers with ordinary addition and multiplication is not a field.

2. (a) In any ordered field F, prove the following properties:

(i) If $z < 0$, then $x < y$ implies $xz > yz$.

(ii) $||x| - |y|| \leq |x - y|$.

(b) Prove that every complete ordered field is Archimedean.

(c) Prove that the sequence $\{x_n\}$, where $x_n = \left(1 + \frac{1}{n}\right)^n$, is convergent.

3. (a) Prove that every convergent sequence of real numbers is bounded. Is the converse true? Justify.

(b) Find the limit of the sequences $\{x_n\}$ whose $n^{th}$ terms are given as follows

(i) $x_n = \frac{n^2+6n}{n^2-5n+1}$

(ii) $x_n = \sqrt{4n^2+n} - 2n$

(iii) $x_n = \frac{n}{3^n}$

(c) State Monotone Convergence theorem. Use it to prove that the sequence $\{x_n\} = \left\{\frac{1}{n}\right\}$ is convergent.



4. (a) Define monotonically increasing and monotonically deceasing sequence. Give an example in each case. Determine the monotonicity of the sequence $\left\{\frac{3^n}{n^3}\right\}$.

(b) Show that if $[a] < 1$, then $\lim_{n\to\infty} a^n = 0$. Hence, evaluate $\lim_{n\to\infty} \left(\frac{-1}{3}\right)^n$.

(c) Prove that every convergent sequence of real numbers is a Cauchy sequence. Further, show that the sequence $\{x_n\}$ defined by $x_n = \frac{n}{n+1}$ is a Cauchy sequence.

5. (a) State Alternating Series Test. Use it to show that the series