

19/5/25

[illegible]

P.T.O.

- (b) The accompanying data consists of observations on the time until failure (1000s of hours) for a sample of turbochargers from one type of engine.

Draw the box plot.

1.6, 2.0, 2.6, 3.0, 3.5, 3.9, 4.5, 4.6, 4.8, 5.0, 5.1, 5.3, 5.4, 5.6, 5.8, 6.0, 6.0, 6.1, 6.3, 6.5

- (c) Poly (3-hydroxybutyrate) (PHB), a semicrystalline polymer that is fully biodegradable and biocompatible, is obtained from renewable resources. Calculate the sample range, mean and variance.

180.5, 181.7, 180.9, 181.6, 182.6, 181.6, 181.3, 182.1, 182.1, 180.3, 181.7, 180.5

2. (a) Consider randomly selecting a student at a large university, and let A be the event that the selected student has a Visa card and B be the analogous event for MasterCard. Suppose that $P(A) = 0.6$ and $P(B) = 0.4$.

(i) Could it be the case that $P(A \cap B) = 0.5$? Why or why not ?

(ii) From now on, suppose that $P(A \cap B) = 0.3$. What is the probability that the selected student has at least one of these two types of cards ?

(iii) What is the probability that the selected student has neither type of card ?

- (b) A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that $P(A) = 0.6$ and $P(B) = 0.05$. What is $P(B \cup A)$?
- (c) An aircraft seam requires 25 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability.
- (i) If 15% of all seams need reworking, what is the probability that a rivet is defective ?
- (ii) How small should the probability of a defective rivet be to ensure that only 10% of all seams need reworking ?
3. (a) Six boxes of components are ready to be shipped by a certain supplier. The number of defective components in each box is as follows :

Box	Number of Defectives
1	0
2	2
3	0
4	1
5	2
6	0

Let X be the number of defectives in the selected box.

- (i) Give the definition of a random variable.
- (ii) Find the probability mass function (pmf) of X .
- (iii) Draw the line graph of pmf.

- (b) The n candidates for a job have been ranked 1, 2, 3, ..., n . Each candidate has an equal chance of being selected for the job. Let the random variable X be defined as X = the rank of a randomly selected candidate.

- (i) Find the probability mass function (pmf) of X .
- (ii) Compute $E(X)$ and $V(X)$.

- (c) For any random variable X , prove that $E(aX + b) = aE(X) + b$ and $\sigma_{aX + b} = |a| \sigma_X$.

4. (a) The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with probability density function (pdf)

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find :

- (i) The cumulative distribution function (cdf) of sales.
 - (ii) σ_x .
- (b) Define Bernoulli distribution. Compute $E(X)$, $E(X^2)$ and $V(X)$ for a Bernoulli random variable X . Is Bernoulli distribution a particular case of Binomial distribution ?

(c) Compute the following binomial probabilities directly from the formula for $b(x : n, p)$:

(i) $b(3 : 8, 0.35)$

(ii) $b(5 : 8, 0.6)$

(iii) $P(3 \leq X \leq 5)$ when $n = 7$ and $p = 0.6$.

5. (a) The direction of an imperfection with respect to a reference line on a circular object such as a tire, brake rotor, or flywheel is, in general, subject to uncertainty. Consider the reference line connecting the valve stem on a tire to the center point, and let X be the angle measured clockwise to the location of an imperfection. One possible pdf for X is

$$f(x) = \begin{cases} \frac{1}{360}, & 0 \leq x < 360 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that the angle is between 90° and 180° . Also, find the probability that the angle of occurrence is within 90° of the reference line.

- (b) The reaction time for an in-traffic response to a brake signal from standard brake lights can be modeled with a normal distribution having mean value 1.25 sec and standard deviation of 0.46 sec. What is the probability that reaction time is between 1.00 sec and 1.75 sec ? And, what is the probability that actual reaction time will exceed 2 sec ?

(c) According to the article "Predictive Model for Pitting Corrosion in Buried Oil and Gas Pipelines" (Corrosion, 2009 : 332-342), the lognormal distribution has been reported as the best option for describing the distribution of maximum pit depth data from cast iron pipes in soil. The authors suggest that a lognormal distribution with $\mu = 0.353$ and $\sigma = 0.754$ is appropriate for maximum pit depth (mm) of buried pipelines. What is the probability that maximum pit depth is between 1 and 2 mm ?

6. (a) The amount of a particular impurity in a batch of a certain chemical product is a random variable with mean value 4.0 g and standard deviation 1.5 g. If 100 batches are independently prepared, what is the (approximate) probability that the sample total amount of impurity T_0 is at most 425 g ?
- (b) Consider the following data on x = iodine value (g) and y = cetane number for a sample of 8 biofuels :

x	y
132	46
129	48
120	51
113	52
105	54
92	52
84	59
83	58

Find the normal equations and hence determine the equation of regression line.

- (c) Consider the following data on corn yield x and peanut yield y (units) for eight different types of soil :

x	y
24	13
34	21
46	18
37	16
22	20
33	17
40	21
21	16

Compute the coefficient of correlation between x and y . And, how would you describe the nature of relationship between the two variables.