

This question paper contains 7 printed pages]

Roll No.

--	--	--	--	--	--	--	--	--	--

S. No. of Question Paper : 7400

Unique Paper Code : 2352203601

Name of the Paper : DSC - Probability and Statistics

Name of the Course : Bachelor of Arts/Bachelor of Science
(Programme) with Mathematics as non-major/
minor

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Consider the following data :

Class	Frequency
[2, 4)	9
[4, 6)	15
[6, 8)	5
[8, 12)	9
[12, 20)	8
[20, 30)	2

P.T.O.

Construct an appropriate histogram and comment on any interesting features.

- (b) The following data consists of observations on the time until failure (1000s of hours) for a sample of turbochargers from one type of engine. Compute the Median, Upper Fourth (third quartile) and Lower Fourth (first quartile)

590, 815, 575, 608, 350, 1285, 408, 540, 555, 679.

- (c) The following data contain the information about fuel characteristics of various vehicles. Find the value of the sample range, mean and variance.

27.3, 27.9, 32.9, 35.2, 44.9, 39.9, 30.0, 29.7, 28.5, 32.0, 37.6.

2. (a) In a certain residential suburb, 60% of all households get Internet service from the local cable company, 80% get television service from that company, and 50% get both services from that company. If a household is randomly selected, what is the probability that it gets at least one of these two services from the company, and what is the probability that it gets exactly one of these services from the company ?
- (b) State Baye's Theorem A large operator of timeshare complexes requires anyone interested in making a purchase to first visit the site of interest. Historical data indicates that 20% of all potential purchasers select a day visit, 50% choose a one-night visit, and 30% opt for a two-night visit. In

addition, 10% of day visitors ultimately make a purchase, 30% of one-night visitors buy a unit, and 20% of those visiting for two nights decide to buy. Suppose a visitor is randomly selected and is found to have made a purchase. How likely is it that this person made a day visit ?

- (c) An aircraft seam requires 25 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability.

(i) If 15% of all seams need reworking, what is the probability that a rivet is defective ?

(ii) How small should the probability of a defective rivet be to ensure that only 10% of all seams need reworking ?

3. (a) Consider whether the next person buying a computer at a certain electronics store buys a laptop or a desktop model.

$$\text{Let } X = \begin{cases} 1, & \text{if the customer purchase a desktop computer} \\ 0, & \text{if the customer purchase a laptop computer} \end{cases}$$

Find the probability mass function (pmf) for X , if 20% of all purchasers during that week select a desktop. Also draw the line graph for the pmf of X .

- (b) A certain brand of upright freezer is available in three different rated capacities : 450L, 500L and 550L. Let X = the rated capacity of a

P.T.O.

freezer of this brand sold at a certain store. Suppose that X has (pmf)
 $p(450) = 0.2$, $p(500) = 0.5$, $p(550) = 0.3$.

(i) Compute $E(X)$, $E(X^2)$ and $V(X)$.

(ii) If the price of a freezer having capacity X is $2.5X - 650$, what is the expected price paid by the next customer to buy a freezer ?

(c) For any random variable X , let $E(X) = 5$ and $E[X(X - 1)] = 27.5$.
Compute :

(i) $E(X^2)$

(ii) $V(X)$

(iii) $V(2X + 3)$.

4. (a) "Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars. Suppose that in a different traffic environment, the distribution of time headway has the form

$$f(x) = \begin{cases} \frac{K}{x^4}, & x > 1 \\ 0, & x \leq 1 \end{cases}$$

(i) Determine the value of K for which $f(x)$ is a pdf.

(ii) Determine $P(X > 2)$.

- (b) Consider the pmf of the magnitude X of a dynamic load on a bridge (in newtons) as

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Find the cumulative distribution function (cdf) for X .
- (ii) Compute $P(1 \leq X \leq 1.5)$ and $P(X > 1)$.
- (c) Define Binomial distribution. Find the mean of a binomial distributed random variable X . When a Bernoulli distribution becomes a particular case of Binomial distribution ?
5. (a) If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is 0.005 and errors are independent from page to page, what are the probabilities that one of its 600-page novels will contain exactly one page with errors and at most three pages with errors ?
- (b) Suppose that 25% of all students at a large public university receive financial aid. Let X be the number of students in a random sample of size 50 who receive financial aid. Find the probability that at most 10 students receive aid. Also, find the probability that between 5 and 15 (inclusive) of the selected students receive aid.

(c) According to the article "Predictive Model for Pitting Corrosion in Buried Oil and Gas Pipelines" (Corrosion, 2009 : 332-342), the lognormal distribution has been reported as the best option for describing the distribution of maximum pit depth data from cast iron pipes in soil. The authors suggest that a lognormal distribution with $\mu = 0.353$ and $\sigma = 0.754$ is appropriate for maximum pit depth (mm) of buried pipelines. What value c is such that only 1% of all specimens have a maximum pit depth exceeding c ?

6. (a) The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation 0.04 cm. If $\bar{X}$ is the sample mean diameter for a random sample of $n = 16$ rings. How likely is it that the sample mean diameter exceeds 12.01 ?
- (b) Consider the following data of on x = rainfall volume (m^3) and y = runoff volume (m^3) for a particular location.

x	y
5	4
12	10
14	13
17	15
23	15
30	25
40	27
47	46
55	38
67	46

Find the equation of regression line. And estimate the runoff volume (y) when rainfall volume (x) is 50.

- (c) Consider the following data on two variables x and y :

x	y
2.4	13
3.4	21
4.6	18
3.7	16
2.2	20
3.3	17
4.0	21
2.1	16

Compute the coefficient of correlation between the variables x and y . Does the value of the coefficient of correlation between the variables change when each value of x and y in the data is doubled.