

This question paper contains 7 printed pages]

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S. No. of Question Paper : 5866

Unique Paper Code : 2352203601

Name of the Paper : DSC-Probability and Statistics

Name of the Course : Bachelor of Arts/Bachelor of Science
(Programme) with Mathematics as non-major/
minor

Semester : VI .

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Consider the following data :

Class	Frequency
[1 3)	1
[3 5)	1

P.T.O.

[5 7)	11
[7 9)	21
[9 11)	25
[11 13)	17
[13 15)	9
[15 17)	4
[17 19)	1

Construct an appropriate histogram and comment on any interesting features.

- (b) Do running times of American movies differ somehow from running times of French movies ? The author investigated this question by randomly selecting 25 recent movies of each type, resulting in the following running times :

94, 90, 95, 93, 128, 95, 125, 91, 104, 116, 162, 102, 90, 110, 92, 113, 116, 90, 97, 103, 95, 120, 109, 91, 138.

Construct a stem-and-leaf plot.

- (c) Calculate the sample mean, variance and standard deviation from the following data :

203, 130, 160, 180, 195, 132, 145, 211, 105, 145, 153, 152, 138, 87, 99, 93, 119, 129.

2. (a) Find the sample space for rolling of two dice and tossing of three coins.
- (b) Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products :
- (i) What is the probability that a randomly selected adult regularly consumes both coffee and soda ?
- (ii) What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products ?
- (c) Consider the following probability :

$$P(A) = 0.6, P(B) = 0.4, P(A \cap B) = 0.3, P(C) = 0.2, P(A \cap C) = 0.15, \\ P(B \cap C) = 0.1, \text{ and } P(A \cap B \cap C) = 0.08.$$

Find the value of $P(A|B)$, $P(B|A)$, $P(A|B \cup C)$.

3. (a) Define Poisson distribution. Find the mean and variance of a Poisson distributed random variable X . Are they equal ?
- (b) Let the random variable be defined as :
- $X = 1$ if a randomly selected vehicle passes an emission test and $X = 0$ otherwise. Assume that the probability mass function (pmf) of this variable is : $p(1) = p$ and $p(0) = 1 - p$.
- (i) Name the random variable.

(ii) Compute $E(X^2)$.

(iii) Show that $V(X) = p(1 - p)$.

(c) Starting at a fixed time, the gender of each newborn child is observed at a certain hospital until a boy (B) is born. Let $p = P(\text{boy is born}) = P(B)$ and assume that the successive births are independent. Let the random variable X = number of births up to and including that of the first boy :

(i) Find the probability mass function (pmf) of X .

(ii) Determine the cumulative distribution function (cdf) of X .

4. (a) Compute the following binomial probabilities directly from the formula for $b(x : n, p)$:

(i) $b(3 : 6, 0.5)$

(ii) $P(3 \leq X)$ when $n = 6$ and $p = 0.5$.

(iii) $P(X \leq 1)$ when $n = 6$ and $p = 0.5$.

(b) Find k such that the function f defined by :

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function (pdf). Also, determine $P(0 < x < 4)$.

(c) Define exponential distribution. Find the mean and standard deviation of an exponential distributed random variable X . Are they equal ?

5. (a) "Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars on a freeway during a period of heavy flow. The following pdf of X is essentially the one suggested in "The Statistical Properties of Freeway Traffic". (Transp. Res., vol. 11 : 221-228) :

$$f(x) = \begin{cases} 0.15e^{-0.15(x-0.5)}, & x \geq 0.5 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that headway time is at most 5 sec.

- (b) Let X , the thickness of a certain metal sheet, have a uniform distribution on $[A, B]$. Write the probability density function (pdf) of X and find the cumulative distribution function (cdf) of X .
- (c) The amount of distilled water dispensed by a certain machine is normally distributed with mean value 64 oz and standard deviation 0.78 oz. What container size c will ensure that overflow occurs only 0.5% of the time?
6. (a) The distribution of egg weights (g) of a certain type is normal with mean value 53 and standard deviation 0.3. Let $X_1, X_2, \dots, X_{12}$ denote the weights of a dozen randomly selected eggs; these X_i 's constitute a random sample of size 12 from the specified normal distribution. Find the mean and standard deviation of the sample mean weight $\bar{X}$. Also find the probability that the sample mean weight exceeds 53.5 g.

- (b) Medical researchers have noted that adolescent females are much more likely to deliver low-birth-weight babies than are adult females. Because such babies have higher mortality rates, numerous investigations have focused on the relationship between mother's age and birth weight. For the following data on x = maternal age (yr) and y = baby's birth weight (kg) :

x	y
15	2.3
17	3.4
18	3.3
15	2.6
16	2.9
19	3.3
17	2.9
16	2.5
18	3.1
19	3.6

Compute the coefficient of correlation between x and y . And, how would you describe the nature of relationship between the two variables.

- (c) Consider the following data on fracture strength (x , as a percentage of ultimate tensile strength) and attenuation (y , in neper/cm, the decrease in amplitude of the stress wave) in fiberglass-reinforced polyester composites :

x	y
12	3.3
30	3.2
36	3.4
40	3.0
45	2.8
57	2.9
62	2.7
67	2.6
71	2.5
78	2.6

Find the equation of regression line. And estimate the attenuation (y) when fracture strength (x) is 50.