This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7452**

Unique Paper Code : **2372572401**

Name of the Paper : **Elements of Statistical Inference**

Name of the Course : **B.Sc. (P)/B.A. (P) with Statistics**

Semester : **IV (NEP)**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *six* questions.

All questions carry equal marks.

Use of non-programable calculator is allowed.

1. (*a*) Let $X_1, X_2, \ldots\ldots X_n$ be a random sample of size n from a large population $X_1, X_2, \ldots\ldots X_N$ of size N with mean μ and variance σ^2. The sample mean and variance is given by, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$. Show that the sample mean is an unbiased estimator of population mean but the sample variance is not an unbiased estimate of population variance. Suggest an unbiased estimator of population variance.

   (*b*) State and prove invariance property of consistency. Show that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator of μ. 8,7

P.T.O.

2\. (*a*) A random sample $(x_1, x_2, ..., x_5)$ of size 5 is drawn from a normal population with mean μ and variance σ^2. Let T_1, T_2 and T_3 are the estimators used to estimate μ, where :

$$T_1 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}, \quad T_2 = \frac{x_1 + x_2}{2} + X_3 \text{ and } T_3 = \frac{(x_1 + x_2 + \lambda x_3)}{3}$$

(*i*) Are T_1 and T_2 unbiased estimators ?

(*ii*) Find the value of λ such that T_3 is unbiased estimator for μ.

(*iii*) Is T_3 a consistent estimator ?

(*iv*) Which is the best estimator among T_1, T_2 and T_3 ?

(*b*) Let T be an MVU estimator of $\gamma(\theta)$ and T_1 and T_2 be two other unbiased estimators of $\gamma(\theta)$ with efficiencies e_1 and e_2 respectively. If ρ_θ is the correlation coefficient between T_1 and T_2, then show that :

$$(e_1 e_2)^{\frac{1}{2}} - \{(1 - e_1)(1 - e_2)\}^{\frac{1}{2}} \le \rho_\theta \le (e_1 e_2)^{\frac{1}{2}} + \{(1 - e_1)(1 - e_2)\}^{\frac{1}{2}}. \qquad 8,7$$

3\. (*a*) State Cramer-Rao inequality. When this inequality becomes equality. Let $X_1, X_2, ..., X_n$ be a random sample from population :

$f(x, \theta) = \frac{1}{\theta} e^{-\left(\frac{x}{\theta}\right)}; 0 < x < \infty.$ Does there exist an MVB estimator for the parameter θ ? If so, find the estimator along with its variance.

(*b*) Let $X_1, X_2, ..., X_n$ be a random sample from $f(x, \theta) = \theta x^{\theta - 1}$, $0 < x < 1, \theta > 0$. Show that $\prod_{i=1}^{n} X_i$ is a sufficient estimator for θ. 8,7

4. (*a*) Define Confidence Intervals for Large Samples. Show that for the distribution $dF(x) = \theta e^{-x\theta}; 0 < x < \infty$, confidence limits for large samples with 95% confidence coefficient are given by $\theta = \left(1 \pm \frac{1.96}{\sqrt{n}}\right)\frac{1}{\bar{x}}$.

(*b*) Let $X_1, X_2, \ldots, X_n$ be a random sample from population $f(x, \theta) = \theta^x (1 - \theta)^{1-x}; x = 0, 1$. Show that $\frac{T(T-1)}{n(n-1)}$ is an unbiased estimator of θ^2, where $T = \sum_{i=1}^{n} X_i$. 8,7

5. (*a*) In random sampling from Normal population $N(\mu, \sigma^2)$, find the maximum likelihood estimators for (*i*) μ when σ^2 is known (*ii*) σ^2 when μ is known (*iii*) Simultaneously μ and σ^2.

(*b*) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test. 8,7

6. (*a*) Define Most Powerful and Uniformly Most Powerful critical regions. Show that a Most Powerful critical region is necessarily unbiased.

(*b*) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a uniform population : $f(x, \theta) = 1; \theta - \frac{1}{2} \leq x \leq \theta + \frac{1}{2}, -\infty < \theta < \infty$. Obtain MLE of θ and comment on the result. 8,7

7. (*a*) Define simple and composite hypothesis. Further, explain best critical region.

(*b*) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$, based on a single observation from the population, $f(x, \theta) = \theta e^{-\theta x}$, $0 \leq x < \infty$, obtain the values of type I error, type II error and power of the test. 7,8

8. Write short notes on any *three* of the following : 3×5=15

(*a*) Optimum properties of maximum likelihood estimators

(*b*) Fisher-Neyman criterion for sufficiency

(*c*) Sufficient conditions for consistency

(*d*) Minimum Variance Unbiased Estimator.