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S. No. of Question Paper : 6948

Unique Paper Code : 2352203601

Name of the Paper : DSC-Probability and Statistics

Name of the Course : Bachelor of Arts/Bachelor of Science
(Programme) with Mathematics as non-major/
minor

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Do running times of American movies differ somehow from running times of French movies ? The author investigated this question by randomly selecting 25 recent movies of each type, resulting in the following running times :

94, 90, 95, 93, 128, 95, 125, 91, 104, 116, 162, 102, 90, 110, 92, 113, 116,
90, 97, 103, 95, 120, 109, 91, 138.

Draw a stem-and-leaf plot.

P.T.O.

- (b) The accompanying data consists of observations from a population :

1.6, 2.0, 2.6, 3.0, 3.5, 3.9, 4.5, 4.6, 4.8, 5.0, 5.1, 5.3, 5.4, 5.6, 5.8, 6.0, 6.0, 6.1, 6.3, 6.5

Construct box plot.

- (c) A mutual fund is a professionally managed investment scheme that pools money from many investors and invests in a variety of securities :

1.03, 1.23, 1.10, 1.64, 1.30, 1.27, 1.25, 0.78, 1.05, 0.64, 0.94, 2.86, 1.05, 0.75, 0.09, 0.79, 1.61, 1.26, 0.93, 0.84

Calculate the sample mean, variance and standard deviation.

2. (a) Let A denote the event that the next request for assistance from a statistical software consultant relates to the SPSS package, and let B be the event that the next request is for help with SAS. Suppose that $P(A) = .30$ and $P(B) = .50$.

(i) Why is it not the case that $P(A) + P(B) = 1$?

(ii) Calculate $P(A')$.

(iii) Calculate $P(A \cup B)$.

(iv) Calculate $P(A' \cap B')$.

- (b) It is known that 30% of a certain company's washing machines require service while under warranty, whereas only 10% of its dryers need such service. If someone purchases both a washer and a dryer made by this company, what is the probability that both machines will need warranty service ?

(c) Consider the following probability :

$$P(A) = 0.6, P(B) = 0.4, P(A \cap B) = 0.3, P(C) = 0.2, P(A \cap C) = 0.15, \\ P(B \cap C) = 0.1, \text{ and } P(A \cap B \cap C) = 0.08.$$

Find the value of $P(A|B)$, $P(B|A)$, $P(A|B \cup C)$.

3. (a) Define Poisson distribution. Find the mean and variance of Poisson distribution. State the conditions when a Poisson distribution becomes a limiting case of Binomial distribution.

(b) A library has an upper limit of 6 on the number of CDs that can be checked out to an individual at one time. Consider only those who currently have CDs checked out. Let X be the number of CDs checked out to a randomly selected individual. The pmf of X as follows :

X	$p(x)$
1	0.30
2	0.25
3	0.15
4	0.05
5	0.10
6	0.15

Find $E(X)$ and $V(X)$.

- (c) For any random variable X , prove that $V(aX + b) = a^2V(X)$ and $\sigma_{aX + b} = |a|\sigma_X$.

4. (a) The weekly demand for propane gas (in 1000s of gallons) from a particular facility is a random variable X with pdf

$$f(x) = \begin{cases} 2\left(1 - \frac{1}{x^2}\right) & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Compute the cumulative distribution function (cdf) of X .
- (ii) Compute $E[X]$ and $V[X]$.
- (b) If X is a binomial distributed random variable with parameter n and p , prove that :
- (i) $E[X] = np$
- (ii) $V[X] = np(1 - p)$.
- (c) "Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars. Suppose that in a different traffic environment, the distribution of time headway has the form :

$$f(x) = \begin{cases} \frac{K}{x^4}, & x > 1 \\ 0, & x \leq 1 \end{cases}$$

- (i) Determine the value of K for which $f(x)$ is a pmf.
- (ii) Obtain the cumulative distribution function (cdf).

5. (a) Suppose the pdf of the magnitude X of a dynamic load on a bridge (in newtons) is given by :

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that the magnitude of a dynamic load is between 0.5 to 1.5 newton. Also, find the probability that the load exceeds 1 newton.

- (b) The authors used data collected from an experiment with a specified initial crack length and number of loading cycles to propose a normal distribution with mean value 5.496 mm and standard deviation 0.067 mm for the random variable X = final crack depth. For this model, what value of final crack depth would be exceeded by only 0.5% of all cracks under these circumstances ?
- (c) The breakdown voltage of a randomly chosen diode of a particular type is known to be normally distributed. What is the probability that a diode's breakdown voltage is within 1 standard deviation of its mean value ? And, what is the probability that a diode's breakdown voltage is within 2 standard deviation of its mean value ?

6. (a) The distribution of egg weights (g) of a certain type is normal with mean value 53 and standard deviation 0.3. Let $X_1, X_2, \dots, X_{12}$ denote the weights of a dozen randomly selected eggs; these X_i 's constitute a random sample of size 12 from the specified normal distribution. Find the mean and standard deviation of the sample total weight T_0 . And, determine the probability that the sample total weight T_0 is between 635 and 640.
- (b) Consider the following data on $y =$ porosity (%) is related to $x =$ unit weight (pcf) in concrete specimens :

x	y
99	29
101	28
102	27
103	25
105	23
107	22
108	21
110	20

Find the normal equations and hence determine the regression line.

- (c) Consider the following data on fracture strength (x , as a percentage of ultimate tensile strength) and attenuation (y , in neper/cm, the decrease in amplitude of the stress wave) in fiberglass-reinforced polyester composites :

x	y
12	3.3
30	3.2
36	3.4
40	3.0
45	2.8
57	2.9
62	2.7
67	2.6
71	2.5
78	2.6

Compute the coefficient of correlation between x and y . And, how would you describe the nature of relationship between the two variables.