

This question paper contains 8 printed pages]

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S. No. of Question Paper : 6961

Unique Paper Code : 2352203601

Name of the Paper : DSC-Probability and Statistics

Name of the Course : Bachelor of Arts/Bachelor of Science
(Programme) with Mathematics as non-major/
minor

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Consider the following data :

2.0	2.4	2.5	2.6	2.6	2.7	2.7	2.8	3.0	3.1	3.2	3.3	3.3
3.4	3.4	3.6	3.6	3.6	3.6	3.7	4.4	4.6	4.7	4.8	5.3	10.1

Find :

(i) Mean,

P.T.O.

(ii) 7.7% trimmed mean,

(iii) 10% trimmed mean.

(b) Consider the following data :

Class	Frequency
[1 3)	1
[3 5)	1
[5 7)	11
[7 9)	21
[9 11)	25
[11 13)	17
[13 15)	9
[15 17)	4
[17 19)	1

Construct an appropriate histogram and comment on any interesting features.

(c) The following data contain the information about fuel characteristics of various vehicles. Find the value of the sample range, mean and variance :

2.75, 2.62, 2.74, 3.85, 2.34, 2.74, 3.93, 4.21, 3.88, 8.33, 3.46, 4.52, 2.43, 3.65, 2.78, 3.56, 3.01.

2. (a) Consider the type of clothes dryer (gas or electric) purchased by each of five different customers at a certain store :
- (i) If the probability that at most one of these purchases an electric dryer is .428, what is the probability that at least two purchase an electric dryer ?
 - (ii) If P (all five purchase gas) = .116 and P (all five purchase electric) = .005, what is the probability that at least one of each type is purchased ?
- (b) Blue Cab operates 15% of the taxis in a certain city, and Green Cab operates the other 85%. After a nighttime hit-and-run accident involving a taxi, an eyewitness said the vehicle was blue. Suppose, though, that under night vision conditions, only 80% of individuals can correctly distinguish between a blue and a green vehicle. What is the (posterior) probability that the taxi at fault was blue ? In answering, be sure to indicate which probability rules you are using.
- (c) A boiler has five identical relief valves. The probability that any particular valve will open on demand is .96. Assuming independent operation of the valves, calculate P (at least one valve opens) and P (at least one valve fails to open).

P.T.O.

3. (a) Define Poisson distribution. Find mean and variance of Poisson distribution.

(b) For any random variable X , prove the following :

(i) $E(aX + b) = aE(X) + b$

(ii) $\text{Var}(aX + b) = a^2\text{Var}(X)$.

(c) In a group of five potential blood donors a, b, c, d and e , only a and b have O positive (O+) blood type. Five blood samples, one from each individual, will be typed in random order until an O+ individual is identified. Let the random variable Y = the number of typing necessary to identify an O+ individual.

(i) Find the probability mass function (pmf) of Y .

(ii) Draw the line graph and probability histogram of the pmf.

4. (a) Find k such that the function f defined by :

$$f(x) = \begin{cases} kx^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function (pdf). Also, determine $P\left(\frac{1}{4} < x \leq \frac{1}{2}\right)$.

- (b) Consider the pdf of the magnitude X of a dynamic load on a bridge (in newtons) as :

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Find the cumulative distribution function (cdf) for X .
- (ii) Compute $P(1 \leq X \leq 1.5)$ and $P(X > 1)$.
- (c) Define exponential distribution. For an exponential distributed random variable X , determine $P(X \leq 10)$ and $P(5 \leq X \leq 10)$ for $\lambda = 6$.
5. (a) The Cal Poly Department of Statistics has a lab with six computers reserved for statistics majors. Let X denote the number of these computers that are in use at a particular time of day. Suppose that the probability distribution of X is as given in the following table; the first row of the table lists the possible X values and the second row gives the probability of each such value :

x	$p(x)$
0	0.05
1	0.10
2	0.15
3	0.25
4	0.20
5	0.15
6	0.10

Find the probability that :

- (i) at most 2 computers are in use,
- (ii) at least 3 computers are in use,
- (iii) between 2 and 5 computers inclusive are in use,
- (iv) the number of computers in use is strictly between 2 and 5.

- (b) Suppose pulses arrive at a counter at an average rate of six per minute.

Find the probability that in a 0.5-min interval at least one pulse is received.

- (c) The weight distribution of parcels sent in a certain manner is normal with mean value 12 lb and standard deviation 3.5 lb. The parcel service wishes to establish a weight value c beyond which there will be a surcharge. What value of c is such that 99% of all parcels are under the surcharge weight ?

6. (a) The amount of a particular impurity in a batch of a certain chemical product is a random variable with mean value 4.0 g and standard deviation 1.5 g. If 50 batches are independently prepared, what is the (approximate) probability that the sample mean amount of impurity $\bar{X}$ is between 3.5 and 3.8 g ?

- (b) Consider the following data on y = porosity (%) is related to x = unit weight (pcf) in concrete specimens :

x	y
110	20
112	17
112	19
113	16
113	17
115	13
115	14
120	11

Find the equation of regression line. And estimate the porosity (y) when unit weight (x) is 100.

- (c) Consider the following data of on x = rainfall volume (m^3) and y = runoff volume (m^3) for a particular location :

x	y
5	4
12	10
14	13

17	15
23	15
30	25
40	27
47	46
55	38
67	46

Compute the coefficient of correlation between x and y . And, how would you describe the nature of relationship between the two variables.