This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6975**

Unique Paper Code : **2352572401**

Name of the Paper : **Abstract Algebra**

Name of the Course : **B.A./B.Sc. (Programme) Mathematics as Non-Major/Minor-DSC**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

Each part carries **7.5** marks.

Use of calculator not allowed.

1. (*a*) Define inverse of an element in a group G with identity e. In each of the following case, find the inverse of element under the given operation :

   (*i*) 13 in Z_{15}

   (*ii*) 13 in U(15)

   (*iii*) $3 - 2i$ in $\mathbf{C}^*$, where $\mathbf{C}^*$ is a group of non-zero complex numbers under multiplication.

P.T.O.

(*b*) State and prove Finite Subgroup Test.

(*c*) State cyclic group. Prove or disprove U(20) is cyclic.

2. (*a*) Define order of an element in a group. Find order of each element in Z_{10}.

(*b*) Define subgroup of a group. Let H and K be subgroups of an Abelian group G. Prove that $HK = \{hk \mid h \in H \text{ and } k \in K\}$ is a subgroup of G.

(*c*) Show that in any group, an element and its inverse have the same order.

3. (*a*) If $\alpha = (1\ 2\ 4\ 5)\ (3\ 6)$ and $\beta = (1\ 2\ 4)\ (1\ 5\ 3)$, then :

(*i*) Show that α and β are even permutations

(*ii*) Find β^{-1}

(*iii*) Find $O(\alpha\beta)$.

(*b*) Show that the factor group $\mathbb{Z}/4\mathbb{Z}$ has an element of order 4.

(*c*) Construct a complete Cayley table for D_4, the group of symmetries of a square. Is D_4 Abelian ? Justify.

4. (*a*) Show that $\phi : Z_{20} \to Z_{20}$ given by $\phi(x) = 4x$ is a group homomorphism.

(*b*) Prove that the factor group $Z_{12}/\langle 3 \rangle$ has no element of order 4.

(*c*) Show that {I, (23)} is not a normal subgroup of S_3, the group of permutation of degree 3.

5. (*a*) Prove that Z_6 is a ring and find its set of units.

(*b*) Let R be a ring and let $S = \{x \in R \mid ax = xa$ for all a in $R\}$. Prove that the set S is a subring of R.

(*c*) If an ideal I of a ring R contains a unit, then show that I = R.

6. (*a*) Define zero-divisors and integral domain. Give an example of a commutative ring without zero-divisors that is not an integral domain.

(*b*) Prove that the characteristic of an integral domain is 0 or prime.

(*c*) Verify that $\varphi : Z_4 \to Z_{12}$ given by $\varphi(x) = 3x$ is a ring homomorphism or not.