This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5874**

Unique Paper Code : **2352572401**

Name of the Paper : **Abstract Algebra**

Name of the Course : **B.A./B.Sc. (Programme)**

Type of the Paper : **DSC**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting *two* parts from each question.

Both parts of a question to be attempted together.

All questions carry equal marks.

Use of calculator is not allowed.

1. (*a*) Define a group. Prove that in a group G, left and right cancellation laws hold true. 2+5.5

(*b*) Show that the set

$$G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \middle| a \in \mathbf{R}, a \neq 0 \right\}$$

is a group under matrix multiplication. 7.5

(c) Describe each symmetry in D_3 (the set of symmetries of an equilateral triangle). Also construct its coresponding Cayley table. 3.5+4

2. (a) Let G be an Abelian group and H, K be subgroups of G. Then prove that :

$$HK = \{hk \mid h \in H, k \in K\}$$

is a subgroup of G. 7.5

(b) Define order of an element in a group G. Find the order of each element in the group U(15). 2+5.5

(c) State and prove One-Step Subgroup Test. 2+5.5

3. (a) Define a cyclic group. Let $|a| = 30$, then find $\langle a^{26}\rangle$, $\langle a^{17}\rangle$, $\langle a^{18}\rangle$ and $|a^{26}|$, $|a^{17}|$ and $|a^{18}|$. 1.5+3+3

(b) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$, $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$

Compute each of the following :

(i) $\alpha\beta$

(ii) $\beta\alpha$

(iii) $o(\alpha)$ and $o(\beta)$. 2+2+3.5

(c) Let $G = S_3$ and $H = \{(1), (13)\}$. Then find left and right cosets of H in G. 7.5

4. (*a*) Let G be group and H be a subgroup of G. Then for a, $b \in$ G, either $a\mathrm{H} = b\mathrm{H}$ or $a\mathrm{H} \cap b\mathrm{H} = \phi$. 7.5

(*b*) Prove that the center Z(G) of a group G is a normal subgroup of G. Let H = {(1), (12)}. Is H normal in S_3 ? 4+3.5

(*c*) Define kernel of a group homomorphism $\phi : G \to \bar{G}$. Show that *Ker* ϕ is a subgroup of group G. 1.5+6

5. (*a*) Find the unity in the ring {0, 2, 4, 6, 8} under addition and multiplication modulo 10. In Z_6, show that $4|2$ (4 divides 2) and in Z_8, show that $3|7$ (3 divides 7). 2.5+5

(*b*) Define characteristic of a ring. Prove that characteristic of an integral domain is either '0' or prime. 2.5+5

(*c*) Show that every non-zero element of Z_n is a unit or a zero divisor. 7.5

6. (*a*) Describe the factor ring Z/6Z. Is Z/6Z a field ? Justify. 3+4.5

(*b*) Determine all ring homomorphisms from Z_n to itself. 7.5

(*c*) Prove that the only ideals of a field F are {0} and F itself. 7.5