This question paper contains **6** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7501**

Unique Paper Code : **2352202402**

Name of the Paper : **Introduction to Graph Theory**

Name of the Course : **B.A.(Prog.) with Mathematics as Major**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

Both parts of a question must be attempted together.

All questions carry equal marks.

1. (*a*) Given the degree sequence [3, 3, 2, 2, 2, 2], determine whether a simple graph with this degree sequence exists. Justify your answer.

   (*b*) Determine whether the following graph has an Euler circuit : Vertices = {A, B, C, D}; Edges = {AB, BC, CD, DA, AC, BD}.

   (*c*) Given two graphs :

   G_1 : Vertices = {A, B, C, D}, Edges = {AB, BC, CD, DA}

   G_2 : Vertices = {W, X, Y, Z}, Edges = {WX, XY, YZ, ZW}

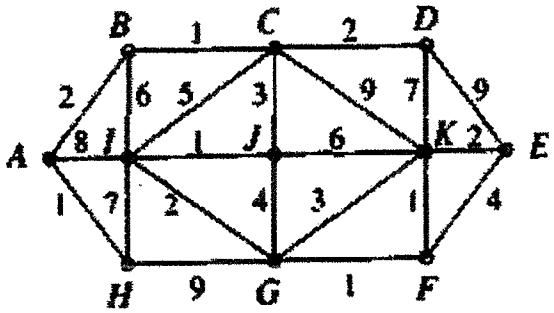
   Are G_1 and G_2 isomorphic ? Justify your answer.

P.T.O.

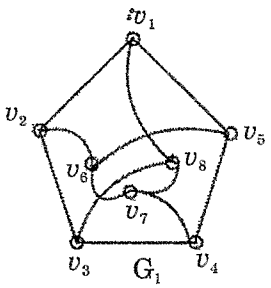
2. (*a*) Determine whether the following graph contains a Hamiltonian cycle :

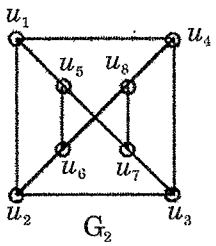
Vertices = {1, 2, 3, 4, 5}; Edges = {12, 23, 34, 45, 51, 13}

(*b*) Apply the original version of Dijkstra's algorithm to find the length of the shortest path from H to every other vertex in the graph. Show the final labels on all vertices. Also find the shortest path from H to D.



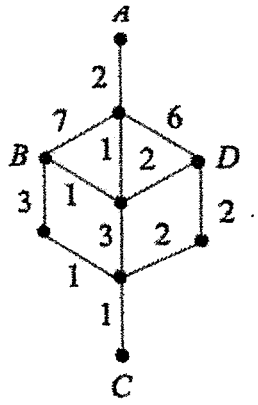
(*c*) (*i*) Find adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 :



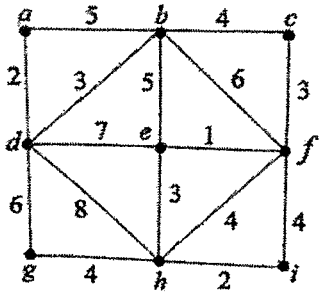


(*ii*) Explain why the function $\phi : G_1 \rightarrow G_2$ defined by $\phi(v_1) = u_1$, $\phi(v_2) = u_2$, $\phi(v_3) = u_6$, $\phi(v_4) = u_8$, $\phi(v_5) = u_4$, $\phi(v_6) = u_3$, $\phi(v_7) = u_7$, $\phi(v_8) = u_5$, is an isomorphism.

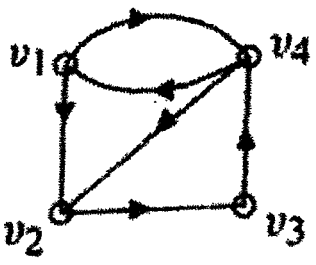
3. (*a*) Solve the Chinese Postman Problem for the following graph :

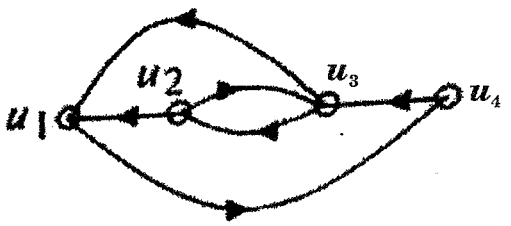


(*b*) Find the minimum spanning tree for the following graph (starting with vertex *a*).

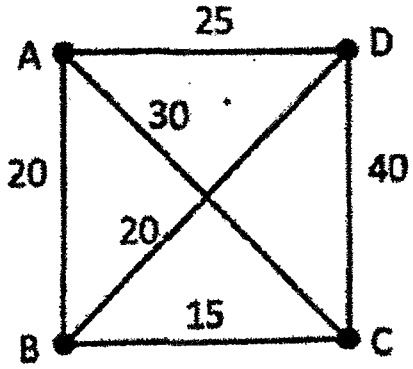


(*c*) Write the definition of a digraph along with an example with 5 vertices. Explain whether the following digraphs are isomorphic or not :

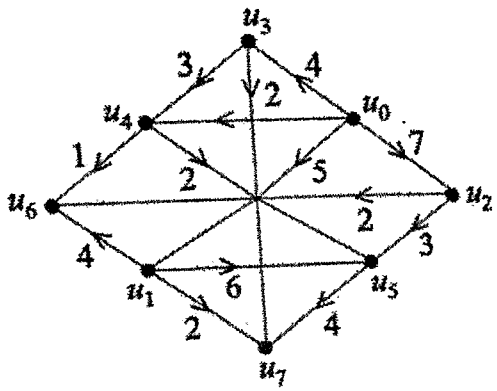




4. (*a*) State the Travelling Salesman's Problem. Solve the same for the following graph, (start at vertex A).



(*b*) Apply Bellman's algorithm to find the lengths of shortest paths from u_1 to each of other vertices :



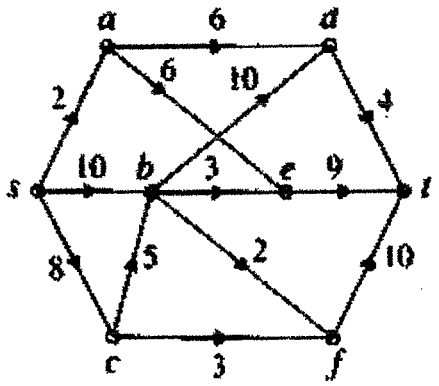
(*c*) State the Kirchhoff Theorem. For the given adjacency matrix of a graph G :

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Draw the graph G and find the total number of spanning trees of G.

5. (*a*) State and prove Max Flow-Min Cut theorem.

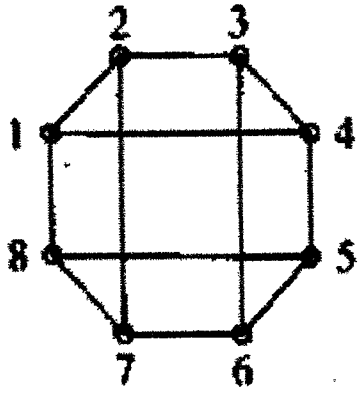
(*b*) Find the maximum flow in the directed network shown in the following figure and prove that it is a maximum :



(*c*) In any directed network, the value of an (s, t)–flow never exceeds the capacity of any (s, t)-cut.

6. (*a*) If G is a bipartite graph with bipartition sets V_1 and V_2, then there exists a matching that saturates V_1 if and only if, for every subset X of V_1, $|X| \leq |A(X)|$.

(*b*) Determine whether the following graph has perfect matching :



(*c*) Let G be Bipartite graph with bipartition sets V_1, V_2 in which every vertex has the same degree k. Show that G has a matching that saturates V_1.