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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6995**

Unique Paper Code : **2352203601**

Name of the Paper : **Probability and Statistics**

Name of the Course : **B.A./B.Sc. (Prog.) Mathematics Non-Major/Minor**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory and

attempt any *two* parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculators

and statistical tables is permitted.

1. (*i*) The **May 1, 2009, issue of The Montclarian** reported the following home sale amounts for a sample homes in Alameda, CA that were sold the previous month (1000s of $) :

590 815 575 608 350 1285 408 540 555 679

(*a*) Calculate and interpret the sample mean and median.

(*b*) Suppose the 6th observation had been 985 rather than 1285. How would the mean and median change ?

P.T.O.

(ii) Zinfandel is a popular red wine varietal produced almost exclusively in California. It is rather controversian among wine connoisseurs because its alcohol content varies quietly substantially from one producer to other. In May 2003, the author went to the website **klwines.com**, randomly selected 10 zinfandels from among the 325 available, and obtained the following values of alcohol content (%) :

14.8 14.5 16.1 14.2 15.9 13.7 16.2 14.6 13.8 15.0

Calculate mean median and sample variance.

(iii) If C_1 and C_2 are subsets of the sample space C, show that :

$$P(C_1 \cap C_2) \leq P(C_1) \leq P(C_1 \cup C_2) \leq P(C_1) + P(C_2).$$

2. (i) A new magazine publishes three columns entitled "Art" (A), "Books" (B) and "Cinema" (C). Reading habits of a randomly selected readers with respect to these columns are :

Read Regularly	**Probability**
A	.14
B	.23
C	.37
$A \cap B$	.08
$A \cap C$	.09
$B \cap C$	.13
$A \cap B \cap C$	.05

Then determine the following probabilities $P(A|B)$, $P(A|B \cap C)$ and $P(A \cup B|C)$.

(*ii*) Given the cdf

$$F(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x+2}{4} & -1 \le x < 1 \\ 1 & 1 \le x \end{cases}$$

Find $P\left(-\frac{1}{2} < X \le \frac{1}{2}\right)$, $P(X = 1)$ and $P(2 < X \le 3)$.

(*iii*) Suppose a random variable X has the pdf

$$f(x) = \begin{cases} kx^2(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find k and $P(1/3 < x < 2)$.

3. (*i*) A certain brand of upright freezer is available in three different rated capacities : 16 ft^3, 18 ft^3 and 20 ft^3. Let X = the rated capacity of a freezer of this brand sold at a certain store. Suppose that X has pmf

X	***p(x)***
16	0.2
18	0.5
20	0.3

(a) Compute E(X) and Var(X).

(b) If the price of a freezer having capacity X is 70X – 650, what is the expected price paid by the next customer to buy a freezer ?

(ii) The current in a certain circuit as measured by an ammeter is a continuous random variable X with the following density function :

$$f(x) = \begin{cases} 0.075x + 0.2, & 3 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

Calculate $P(X \leq 4)$, $P(3.5 \leq X \leq 4.5)$ and $P(4.5 < X)$.

(iii) Define Binomial distribution and calculate its mean and variance.

4. (i) Show that in the limit when $n \to \infty$, $\theta \to 0$ and $n\theta = \lambda > 0$ remains constant, the number of successes in the binomial distribution is a random variable having Poisson distribution with parameter λ.

(ii) Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let X denote the number among 15 randomly selected copies that fail the test. Assume X has a binomial distribution that calculate :

(a) The probability that at most 8 fail the test.

(b) The probability that exactly 5 fail the test.

(iii) Five individuals from an animal population thought to be near extinction in a certain region have been caught, tagged, and released to mix into the population. After they have had an apportunity to mix, a random sample of 10 of these animals is selected. Let X = the number of tagged animals in the second sample. Suppose there are actually 25 animals of this type in the region. Find :

(*a*) The probability that exactly two of the animals in the second sample are tagged.

(*b*) The probability that at most four of the animals in the second sample are tagged.

5. (*i*) Suppose the force acting on a column that helps to support a building is a normally distributed random variable X with mean value 15.0 kips and standard deviation 1.25 kips. Compute the following probabilities by standardizing the variable X :

$$P(X \leq 15),\ P(X \geq 10),\ \text{and}\ P(|X - 15|) \leq 3.$$

(*ii*) The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviatgion 0.04 cm. If $\overline{X}$ is the sample mean diameter for a random sample of $n = 16$ rings :

(*a*) Where is the sampling distribution of $\overline{X}$ centered ?

(*b*) What is standard deviation of the $\overline{X}$ distribution ?

(*c*) How likely is it that the sample mean diameter exceeds 12.01 ?

(*iii*) State central limit theorem. The amount of a particular impurity in a batch of a certain chemical product is a random variable with mean value

4.0 g and standard deviation 1.5 g. If $\overline{X}$ is the sample mean impurity for a random sample of 50 batches.

(a) What is the standard deviation of the $\overline{X}$ distribution ?

(b) What is the probability that the sample average amount of impurity $\overline{X}$ is between 3.5 and 3.8 g ?

6. (i) There are 40 students in an elementary statistics class. On the basis of years of experience, the instructor knows that the time needed to grade a randomly chosen first examination paper is a random variable with an expected value of 6 min and a standard deviation of 6 min.

(a) If grading times are independent and the instructor begins grading at 6:50 p.m. and grades continuously, what is the (approximate) probability that he is through grading before the 11:00 p.m. TV news begins ?

(b) If the sports report begins at 11:10, what is the probability that he misses part of the report if he waits until grading is done before turning on the TV ?

(ii) The following table gives the data on x = rainfull volume (m^3) and y = runoff volume (m^3) for a particular location :

x	y
5	4
12	10
14	13

17	15
23	15
30	25
40	27
47	46
55	38
67	46
72	53
81	70
96	82
112	99
127	100

Determine the equation of the estimated regression line using the principle of least square.

(*iii*) The Turbine Oil Oxidation Test (TOST) and the Rotating Bomb Oxidation Test (RBOT) are two different procedures for evaluating the oxidation stability of steam turbine oils. The accompanying observations on x = TOST time (hour) and y = ROBT time (minute) for 12 oil specimens.

TOST	**RBOT**
4200	370

3600	340
3750	375
3675	310
4050	350
2770	200
4870	400
4500	375
3450	285
2700	225
3750	345
3300	285

(*a*) Calculate the value of the sample correlation coefficient.

(*b*) How would the value of r be affected if we had let x = RBOT time and Y = TOST time ?