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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7020**

Unique Paper Code : **2352572401**

Name of the Paper : **Abstract Algebra**

Name of the Course : **B.A./B.Sc. (Programme)**

Type of the Paper : **DSC**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting *two* parts from each question.

Both parts of a question to be attempted together.

All questions carry equal marks.

Use of Calculator is not allowed.

1. (*a*) Show that GL(2, **R**) (general linear group of 2×2 matrices over **R**) is a group under matrix multiplication. 7.5

(*b*) Prove that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$. 7.5

(*c*) Describe each symmetry in D_4 (the set of symmetries of a square) and write inverse of each element in D_4. 7.5

2. (*a*) Let G be an Abelian group under multiplication with identity e. Then prove that :

$$H = \{x \in G \mid x^2 = e\}$$

is a subgroup of G. 7.5

(*b*) Define Z(G), center of a group G. Show that Z(G) is a subgroup of G. 2+5.5

(*c*) Find the order and inverse of each element in the group U(10). 7.5

3. (*a*) List the elements of the subgroup <3> and <15> in Z_{18}. Let a be a group element of order 18. List the element of the subgroups $\langle a^3 \rangle$ and $\langle a^{15} \rangle$. 4+3.5

(*b*) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}, \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$

Compute each of the following :

(*i*) α^{-1} and β^{-1}

(*ii*) Is $\alpha\beta = \beta\alpha$

(*iii*) $o(\alpha)$ and $o(\beta)$ 2+3+2.5

(*c*) Find all the left and right cosets of {1, 11} in U(30). 7.5

4. (*a*) Let H be a normal subgroup of a group G and K be any subgroup of G. Then $HK = \{hk \mid h \in H, k \in K\}$ is subgroup of G. 7.5

(*b*) Let G = Z and H = 4Z. Construct the Cayley table for the factor group Z/4Z. (7.5)

(*c*) Let $\varphi : G \to \overline{G}$ be a group homomorphism. If H is normal in G, then prove that $\varphi(H)$ is normal in $\overline{G}$. Prove that the mapping $\varphi : (\mathbf{R} - \{0\}, \times) \to (\mathbf{R} - \{0\}, \times)$ such that $\varphi(x) = x^2$ is a homomorphism; where '×' denotes usual multiplication among real numbers. 4.5+3

5. (*a*) Let $R = \left\{ \begin{bmatrix} a & a \\ b & b \end{bmatrix} \middle| a, b \in Z \right\}$. Prove or disprove that R is a subring of $M_2(Z)$. Is $2Z \cup 3Z$ subring of Z ? 5+2.5

(*b*) Prove that if a ring has a unity, then it is unique. Give an example of ring elements a, b such that $ab = 0$, but $ba \neq 0$. 3+4.5

(*c*) Define an Integral Domain. Prove that a finite integral domain is a field. 2.5+5

6. (*a*) Determine all ring homomorphisms from Z to Z. 7.5

(*b*) Let $S = \{a + bi \mid a, b \in Z, b \text{ is even}\}$. Show that S is a subring of $Z[i]$ but not an ideal of $Z[i]$. 4.5+3

(*c*) Describe the factor ring Z/5Z. Is Z/5Z a field ? Justify. 3+4.5