(b) Define center of a ring. Prove that it is a subring. 2.5+5=7.5

(c) Intersection of two subrings is a subring but union may not be. Justify this statement. 4.5+3=7.5

6. (a) In Z_7, give a reasonable interpretation of $\frac{1}{2}$ and $-\frac{2}{3}$. 7.5

(b) Is the ring 2Z isomorphic to the ring 3Z ? Justify. 7.5

(c) Let $S=\left\{\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \middle| a,b \in \mathbb{R}\right\}$, show that $f:\mathbb{C}\to S$ given by $f(a+ib)=\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is a ring isomorphism. 7.5

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[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7029** **I**

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A./B.Sc. (Programme) DSC**

Semester : IV

Time : 3 Hours ***Maximum Marks : 90***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) Attempt **all** questions by selecting **two** Parts from each question.

(iii) Both parts of a question to be attempted together.

(iv) **All** questions carry equal marks.

(v) Use of Calculator is not allowed.

1. (a) Define an Abelian group. Prove that in a group G if ab = ca implies b = c for all a, b, c in G then G is Abelian (that is, left-right cancellation property implies Abelian). 2+5.5=7.5

(b) Show that the set $\mathbb{R}^*$, the set of non-zero real numbers, is a group under multiplication. 7.5

(c) Construct the Cayley table of the Dihedral group D_4 (the group of symmetries of a square). 7.5

2. (a) Let G be an Abelian group under multiplication with identity e. Then prove that

$$H = \{x^2 \mid x \in G\}$$

is a subgroup of G. 7.5

(b) Define order of an element in a group G. Find the order and inverse of each element in the group U(12). 2+5.5=7.5

(c) State and prove Finite Subgroup Test.

2+5.5=7.5

3. (a) Let G be group and let a belong to G. If a has infinite order, then $a^i = a^j$ if and only if $i = j$. If a has finite order, say, n, then $< a > = \{e, a, a^2, a^3, \ldots\ldots, a^{n-1}\}$ and $a^i = a^j$ if and only if n divides $i - j$. 3+4.5=7.5

(b) Let S_3 denotes the set of all one-to-one and onto functions from {1, 2, 3} to itself. How many elements are in S_3 ? List them.

1.5+6=7.5

(c) Let $G = Z_9$ and $H = \{0, 3, 6\}$. Find left and right cosets of H in G. 7.5

4. (a) Define index of a subgroup in a group. Let $|a| = 30$ How many left cosets of $< a^4 >$ in $< a >$ are there ? 2+5.5=7.5

(b) Let H = {(1), (12) (34)} in A_4. Show that H is not normal in A_4. 7.5

(c) Show that the mapping φ from $\mathbb{R}^*$ to $\mathbb{R}^*$, defined as $\varphi(x) = |x|$ where $\mathbb{R}^*$ denotes set of non-zero real numbers, is a homomorphism. Also find Ker φ. 4.5+3=7.5

5. (a) Give an example of finite non commutative ring and of infinite non commutative ring that does not have a unity. Show by an example that for fixed elements a and b in a ring, the equation ax = b can have more than one solution, justify it with a reason.

1.5+2+4=7.5