This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 4736

Unique Paper Code : 2352572301

Name of the Paper : **Differential Equations**

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor**

Type of the Paper : **DSC-B2**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of simple calculator is allowed.

1. (*a*) Solve the differential equation

$$\frac{dy}{dx} = 2y - y^3, \; y(0) = 1. \qquad 7.5$$

(*b*) Show that $y_1 = e^x$ and $y_2 = e^{2x}$ are the solutions of the differential equation

$$y'' - 3y' + 2y = 0.$$

Also, verify that y_1 and y_2 are linearly independent. 7.5

(*c*) Solve the following exact differential equation :

$$(x + y)^2 dx - (y^2 - 2xy - x^2)\, dy = 0. \qquad 7.5$$

2. (*a*) Find the orthogonal trajectories of the family of curves $y = ce^{-3x}$, where c is a real parameter. 7.5

(*b*) Assume that the population of a certain country increases at a rate proportional to the number of inhabitants at any time. If the population doubles in 50 years, in how many years will it triple ? 7.5

(*c*) Define the Wronskian of solutions of second order differential equation. Suppose y_1 and y_2 are the solutions of a second order differential equation. Show that y_1 and y_2 are linearly independent if and only if the Wronskian $W(x) \neq 0$ for every x in the domain. 7.5

3. (*a*) Find the general solution of the differential equation by using the method of undetermined coefficients

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \cos(x) + xe^{-x}.$$ 7.5

(*b*) Solve the initial value problem :

$$y'' + y' - 6y = 0,\ y(0) = 1,\ y'(0) = -7.$$ 7.5

(*c*) Find the general solution of the differential equation by using the method of variation of parameter :

$$y'' - 3y' + 2y = \sin(3x).$$ 7.5

4. (*a*) Find the general solution of the given differential equation :

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3.$$ 7.5

(*b*) Write the general solution of the 8th-order homogeneous differential equation whose auxiliary equation has the following roots :

$$0, 3, 3, -3, 1+2i, 1-2i, 1+i, 1-i.$$

Verify that e^{-x} and e^{2x} are solutions of the homogeneous differential equation.

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0.$$

Show that a linear combination of e^{-x} and e^{2x} is also a solution. 7.5

(*c*) Find the general solution of the linear system :

$$\frac{dx}{dt}+\frac{dy}{dt}-x-3y=3t,$$

$$\frac{dx}{dt}+2\frac{dy}{dt}-2x-3y=1.$$ 7.5

5. (*a*) Eliminate the arbitrary function f from the equation :

$$z = e^{2x} f(x+y),$$

and obtain the partial differential equation. Also, specify the order. 7.5

(*b*) Find the general solution of the Cauchy problem for the first-order PDE :

$$y\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial y}=0, \quad u(0, y) = e^{-y^2}.$$ 7.5

(*c*) Reduce the equation

$$u_x + xu_y = y,$$

to canonical form and obtain the general solution. 7.5

6. (*a*) Use the Method of Characteristics to find the general solution of the equation :

$$uy^2u_x + ux^2u_y = xy^2.$$ 7.5

(*b*) Apply the method of separation of variables $u(x, y) = X(x)\ Y(y)$ to solve

$$au_x + bu_y = 0,\ u(x,\ 0) = ae^{\beta x}.$$ 7.5

(*c*) Classify the partial differential equation as elliptic, parabolic or hyperbolic :

$$u_{xx} = -\ x^2 . u_{yy},\quad x \neq 0.$$

Also, reduce it to canonical form. 7.5