5. (a) State Leibnitz test for convergence of an alternating series of real numbers $\sum_{n=1}^{\infty}(-1)^n u_n$, where $u_n > 0$.

Apply it to test the convergence and absolute convergence of the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$$

(b) If $\sum_{n=1}^{\infty} a_n$ is a positive term convergent series, then show that $\sum_{n=1}^{\infty}\frac{a_n}{n}$ is convergent.

(c) Test the convergence of the series $\sum\frac{(-1)^n \sin n\alpha}{n^3}$, α being real.

6. (a) State D'Alembert's Ratio test for the convergence of positive term series. Use it to test the convergence of the series $\sum_{n=1}^{\infty}\frac{2^{n-1}}{3^n+1}$.

(b) Prove that if the series Σu_n converges then $\lim_{n\to\infty} u_n = 0$. Show by an example that the converse is not true.

(c) State Integral test for the convergence of a positive term series. Use it to examine the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$.

Your Roll No..............

Sr. No. of Question Paper : 2271 **I**

Unique Paper Code : 2352203501

Name of the Paper : Elements of Real Analysis

Name of the Course : **B.A. / B.Sc. (P) with Mathematics as Non-Major / Minor**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) Define supremum and infimum of non-empty subsets of $\mathbb{R}$. Find sup and inf of the following sets :

(i) $S=\left\{\frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$

(ii) $T = \left\{\frac{6n+3}{n} : n \in \mathbb{N}\right\}$

(iii) $U = \left\{2, \frac{3}{2}, \frac{4}{3}, \ldots, \frac{n+1}{n}, \ldots\right\}$.

(b) Define bounded and unbounded subsets of $\mathbb{R}$. Give two examples of each :

(i) Bounded above but not bounded below

(ii) Bounded below but not bounded above

(iii) Neither bounded above nor bounded below

(iv) Both bounded above and bounded below.

(c) Let A and B be bounded non-empty subsets of $\mathbb{R}$ and let

$$A + B = \{a + b : a \in A,\ b \in B\}.$$

Show that sup (A + B) = sup A + sup B.

2. (a) Show that $\forall$ x, y $\in \mathbb{R}$

$$\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}.$$

(b) Prove that a sequence can not converge to more than one real number.

(c) Show that if $\{x_n\}$ is a convergent sequence and $c \in \mathbb{R}$, then $\lim\limits_{n\to\infty} cx_n = c \lim\limits_{n\to\infty} x_n$.

3. (a) Define the convergence of a sequence $\{x_n\}$. Use it to prove that $\lim\limits_{n\to\infty} \frac{3n^2-4n}{n^2+5} = 3$.

(b) Prove that if $\{a_n\}$ is a bounded sequence ad $\{b_n\}$ converges to 0, then $\{a_n b_n\} \to 0$. Use this result to prove that $\lim\limits_{n\to\infty} \frac{\cos n}{n} = 0$..

(c) Use Algebra of Limits to find the following limits :

(i) $\lim\limits_{n\to\infty} \left(\frac{5-2n}{1+6n}\right)^3$

(ii) $\lim\limits_{n\to\infty} \frac{\sqrt{1+n^2}}{1-n}$

4. (a) Prove that every convergent sequence is bounded. Is the converse true? Explain.

(b) For fixed c > 0, prove that $\lim\limits_{n\to\infty} c^{1/n} = 1$.

(c) Consider the sequence $\{x_n\}$ defined inductively by $x_1 = 1$ and $\forall$ n $\in \mathbb{N}$

$$x_{n+1} = \sqrt{4x_n + 5}\ .$$

Prove that $\{x_n\}$ converges and find its limit.