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S. No. of Question Paper : 2564

Unique Paper Code : 2352571101

Name of the Paper : DSC : Topics in Calculus

Name of the Course : B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor

Semester : I

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting two parts from each question.

All questions carry equal marks.

1. (a) Show that the function $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous but not differentiable

at $x = 0$.

- (b) Find the n th differential coefficients of $\sin 3x \sin 2x$.

- (c) If $u = x^3 + y^3 + z^3 + 3xyz$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 3u$.

2. (a) Test the continuity of a function at $x = 0$ which is defined as follows :

$$f(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

P.T.O.

(b) If $y = e^{a \sin^{-1} x}$ (or $x = \sin \left[\frac{(\log y)}{a} \right]$),

prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$.

(c) If $u = \tan^{-1} \frac{xy}{\sqrt{1 + x^2 + y^2}}$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{(1 + x^2 + y^2)^{\frac{3}{2}}}$.

3. (a) State Taylor's theorem with Lagrange's form of remainder. Find the Taylor series expansion of $f(x) = \sin x$.

(b) Evaluate : $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$.

- (c) Discuss the applicability of Rolle's theorem for $f(x) = \tan x$ in $[0, \pi]$.

4. (a) If in Cauchy's mean value theorem, $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$, then show that c is the geometric mean of a and b .

- (b) Verify Rolle's theorem for :

(i) $f(x) = (x^2 + 2x - 3)e^x$ $x \in [-3, 1]$

(ii) $f(x) = 10x - x^2$ $x \in [0, 10]$.

- (c) With the help of Maclaurin's theorem give the expansion of $f(x) = \cos x$ in ascending power of x .

5. (a) Find all the asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1.$$

- (b) Find the range of values of x in which the curve :

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7.$$

is concave upwards or downwards. Also find its points of inflexion.

- (c) Find the reduction formula for

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx.$$

Hence, evaluate $\int_0^{\frac{\pi}{2}} \cos^6 x \, dx$.

6. (a) Determine the position and nature of the double points on the curve :

$$y^2 = (x - 2)^2 (x - 1).$$

- (b) Trace the curve $y = x^3$.

- (c) Obtain a reduction formula for $\int \sin^m x \cos^n x \, dx$. Hence evaluate

$$\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x \, dx.$$