$$u_{xx} - 4\,u_{xy} + 4u_{yy} = e^y.$$

(7.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4312 **I**

Unique Paper Code : 2354002001

Name of the Paper : Differential Equations

Name of the Course : **COMMON PROG GROUPS (Generic Elective)**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting two parts from each question.

3. Both parts of the question to be attempted together.

4. All questions carry equal marks.

5. The use of a calculator is not allowed.

P.T.O.

1. (a) Show that the function defined by $f(x) = (2x^2 + 2e^{3x}+3)e^{-2x}$ satisfies the differential equation

$$\frac{dy}{dx} + 2y = 6e^x + 4xe^{-2x}$$

and also the condition f(0) = 5. (7.5)

(b) Solve $(2xy - \sin x)\, dx + (x^2 - \cos y)\, dy = 0$ (7.5)

(c) Solve the initial-value problem

$$x \sin y\, dx + (x^2 + 1) \cos y\, dy = 0;$$

$$y(1) = \frac{\pi}{2}$$ (7.5)

$$u_x - yu_y - u = 1.$$ (7.5)

6. (a) Using $V = \ln u$ and $V(x, y) = f(x) + g(y)$ to solve the equation

$$x^2 u_x{}^2 + y^2 u_y{}^2 = u^2.$$ (7.5)

(b) Find the solution of the initial value systems

$$u_t - 2\, uu_x = v - x, \quad v_t + av_x = 0$$

with $u(x, 0) = x$ and $v(x, 0) = x$. (7.5)

(c) Obtain the general solution of

$$\frac{dx}{dt} = 5x + 3y, \ \frac{dy}{dt} = 4x + y.$$

Write the general solution. (7.5)

5. (a) Find the solution of the equation $u_x - u_y = 1$ with Cauchy data $u(x, 0) = x^2$. (7.5)

(b) Find the general solution of the equation

$$xu_x + yu_y + zu_z = au + \frac{xy}{z}. \qquad (7.5)$$

(c) Reduce the following equation into canonical form and find the general solution

2. (a) Solve the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = xy^2. \qquad (7.5)$$

(b) Solve the differential equation $(x^2 + y^2 + 1)\, dx - 2xy\, dy = 0$ by first finding an integrating factor. (7.5)

(c) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$. (7.5)

3. (a) Solve the initial value problem

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0, \qquad y(0) = 1, \ y'(0) = 6.$$

(7.5)

(b) Use the method of undetermined coefficients to find the general solution of the differential equation (7.5)

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{3x}.$$

(c) Use the method of variation of parameters to find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = \tan x \sec x. \quad (7.5)$$

4. (a) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 4\ln x. \quad (7.5)$$

(b) Solve the linear system

$$\begin{aligned} 2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x &= t \\ 2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y &= 2. \end{aligned} \quad (7.5)$$

(c) Show that $x = 3e^{7t}, y = 2e^{7t}$ and $x = e^{-t}, \ y = -2e^{-t}$ are two linearly independent solutions on every interval $a \le t \le b$ of the homogeneous linear system