

(b) Show that $\int \cos^3 x \, dx = \frac{3}{4} \sin x + \frac{1}{12} \sin 3x$.

8. (a) Discuss the derivability of the function :

$$f(x) = \begin{cases} x & , x < 1 \\ 2 - x & , 1 \leq x \leq 2 \\ -2 + 3x - x^2 & , x > 2 \end{cases}$$

at $x = 1$ and 2 .

(b) By the elimination of the constants h and k , find the differential equation for which $(x - h)^2 + (y - k)^2 = a^2$, is a solution.

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1405

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Unique Paper Code : 2372011103

Name of the Paper : Calculus

Name of the Course : B.Sc. (Hons) Statistics (NEP-UGCF)

Semester : V

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt six questions in all.
3. All questions/parts carry equal marks.

1. (a) Show that the function f , defined by

$f(x) = (1 + 3x)^{1/x}$ when $x \neq 0$, $f(0) = e^3$, is continuous for $x = 0$.

- (b) If $y = x \log \frac{x-1}{x+1}$, prove that

$$\frac{d^n y}{dx^n} = (-1)^n (n-2)! \left[\frac{(x-n)}{(x-1)^n} - \frac{(x+n)}{(x+1)^n} \right]$$

2. (a) Determine $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \cot^2 x \right)$.

- (b) If A, B, C are the angles of a triangle such that

$\sin^2 A + \sin^2 B + \sin^2 C = \text{Constant}$, prove that

$$\frac{dA}{dB} = \frac{\tan A - \tan B}{\tan C - \tan A}$$

3. (a) Evaluate $\iint xy(x+y) dx dy$ over the area between $y = x^2$ and $y = x$.

- (b) Solve $x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1$.

4. (a) Solve the partial differential equation

$$p(q^2 + 1) + (b - z)q = 0.$$

- (b) Show that $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta = \frac{1}{2} \beta \left(\frac{3}{4}, \frac{1}{4} \right) = 2 \int_0^\infty \frac{x^2}{1+x^4} dx$.

5. (a) Change the order of integration in the integral

$$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{ax}} f(x, y) dx dy.$$

- (b) Find $y_n(0)$ when $y = \log(x + \sqrt{1+x^2})$.

6. (a) Assuming the validity of differentiation under integral sign, show that

$$\int_0^\infty e^{-x^2} \cos ax dx = \frac{1}{2} \sqrt{\pi} e^{-\frac{1}{4}a^2}.$$

- (b) Solve the partial differential equation,

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y.$$

7. (a) Solve $(D^3 - 2D + 4)y = x^4 + 3x^2 - 5x + 2$.