

What is the state probability vector after 2 days?

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1394

I

Unique Paper Code : 2342011103

Name of the Paper : Mathematics for Computing

Name of the Course : B.Sc (H) Computer Science

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Question No. 1 is compulsory.
3. Attempt any four of Question nos. 2 to 7
4. Parts of a Question must be answered together.

1. (a) Find the dot product, cross product and angle between the vector $\vec{a} = \hat{i} + 5\hat{j} - 2\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + 3\hat{k}$

(5)

(b) Is $Q(x) = 6x_1^2 + 3x_2^2 + x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3$ is positive definite ?

(5)

(c) Define bases of vector space. Check whether $A = \{[1,0,0], [0,1,1], [1,1,1]\}$ is a bases of vector space $\mathbb{R}^3$ or not?

(5)

(d) Find Rank of the following matrix using reduced row echelon form

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix} \quad (5)$$

(e) Determine whether $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that $f([x_1, x_2, x_3]) = [x_2, x_3, x_1]$ is linear transformation or not.

(5)

(S), Rainy (R) and Cloudy (C) with transition matrix

(1+3+3)

$$\begin{array}{c} S \\ R \\ C \end{array} \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

S R C

(i) If today is **Sunny**, what is the probability that it will be **Cloudy** tomorrow?

If today is **Rainy**, what is the probability that it will be **Sunny** after two days?

(iii) If the initial state vector is:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

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6. (a) Apply Gram Schmidt orthonormalization process to obtain an orthonormal bases for given bases of $\mathbb{R}^3 : \{[1,0,-1],[-1,4,-1],[2,1,2]\}$ (8)

- (b) Find inverse of the following matrix using row echelon form.

$$\begin{bmatrix} 2 & -6 & 5 \\ -4 & 12 & -9 \\ 2 & -9 & 8 \end{bmatrix} \quad (7)$$

7. (a) Calculate $\text{grad}(\text{div}(\text{curl } \vec{F}))$ of the following vector field

$$\vec{F} = x^3y^3z\hat{i} + x^2y^3z^4\hat{j} + xyz\hat{k} \quad (8)$$

- (b) A weather model uses a Markov chain to predict daily weather based on the states Sunny

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- (f) Find the directional derivative of $F(x,y,z) = 4x^2 + y^2 + 3z^2$ at $P(3,2,4)$ in the direction $5\hat{i} + 6\hat{k}$. (5)

- 2 (a) The set $\mathbb{R}^2$ defined with the addition operation $[x,y] \oplus [w,z] = [x+w-2, y+z+3]$ and scalar multiplication

$$a \odot [x,y] = [ax - 2a + 2, ay + 3a - 3].$$

Show that $\mathbb{R}^2$ is a vector space over addition and scalar multiplication. (8)

- (b) Define inner product space. Consider a real vector space $\mathbb{R}^2$, which is defined as $\langle x, y \rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + 2x_2 y_2$. Show that it is inner product space. (7)

P.T.O.

3. (a) Solve using the Gauss Jordan Method

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 16 \\ 3x_1 + 2x_2 + x_4 &= 16 \\ 2x_1 + 12x_2 - 5x_4 &= 5 \end{aligned} \quad (8)$$

- (b) Find the bases of row space and null space of the following matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix} \quad (7)$$

4. (a) Solve the following set of equations using Gauss Elimination method.

$$\begin{aligned} 5x - 5y - 15z &= 40 \\ 4x - 2y - 6z &= 19 \\ 3x - 6y - 17z &= 41 \end{aligned} \quad (8)$$

- (b) Find value(s) of λ for which following system of equations is consistent.

$$\begin{aligned} 2x + 3y &= 4 \\ x + y + z &= 4 \\ x + 2y - z &= \lambda \end{aligned} \quad (7)$$

5. (a) Diagonalize the following matrix (8)

$$\begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$$

- (b) Define Cayley-Hamilton theorem and verify it for the following matrix

$$\begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix} \quad (7)$$