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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **4535**

Unique Paper Code : **2352203501**

Name of the Paper : **Elements of Real Analysis (DSC-B5)**

Name of the Course : **B.A. (Prog.)/B.Sc. (Prog.)**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory and attempt any *two* parts from each question.

1. (*a*) Define field and check whether or not set of natural numbers **N** is a field with respect to ordinary addition and multiplication. 7.5

(*b*) State Archimedean property of real numbers. Find upper bound and lower bound of the following sets : 7.5

(*i*) $\left\{\frac{1}{x} : 1 < x < 2\right\}$

(*ii*) $\left\{2 - \frac{1}{n} : n \in \mathbf{N}\right\}$.

(*c*) Define bounded set. Find the supremum and infimum of $X = \left\{\frac{1}{n} : n \in \mathbf{N}\right\}$. 7.5

2. (*a*) Define convergence of sequence and prove that a sequence cannot converge to more than one real number. 7.5

P.T.O.

(b) Consider the sequence $x_1 = 1, x_{n+1} = \frac{2x_n + 3}{7}$ for all $n \geq 1$. Prove that $\{x_n\}$ converges and find its limit. 7.5

(c) State First Squeeze principle and using it find the limit of the following sequences : 7.5

(i) $\left\{\frac{(-1)^n}{n}\right\}$

(ii) $\left\{\frac{\sin n}{n^2}\right\}$.

3. (a) Use second squeeze principle to prove that, If $|a| < 1$, then $\lim_{n\to\infty} a^n = 0$. 7.5

(b) Suppose $\{x_n\}$ and $\{y_n\}$ are convergent sequences. Then show that : 7.5

$$\lim_{n\to\infty}(x_n + y_n) = \lim_{n\to\infty} x_n + \lim_{n\to\infty} y_n.$$

(c) Find the limit of the sequence $\{n^{1/n}\}$. 7.5

4. (a) Show that every Cauchy sequence is bounded. Is the converse true ? Justify. 7.5

(b) Check if the following sequences are Cauchy or not : 7.5

(i) $\left\{\frac{n}{n^2+1}\right\}$

(ii) $\left\{\frac{n^2+1}{n}\right\}$.

(c) State necessary conditions for convergence of series. Check the convergence of the following series : 7.5

(i) $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+11}$

(ii) $\sum_{n=1}^{\infty} \frac{n}{n+1}$.

5. (a) Test the convergence of the following series : 7.5

(i) $\sum_{n=1}^{\infty} \frac{n^3}{3n}$

(ii) $\frac{1}{3}+\frac{\sqrt{2}}{5}+\frac{\sqrt{3}}{7}+\frac{\sqrt{4}}{9}+\frac{\sqrt{5}}{11}$.........

(b) Check for absolute convergence of series : 7.5

$$1-\frac{1}{2^2}+\frac{1}{3^3}-\frac{1}{4^4}+\frac{1}{5^5}\text{.........}$$

(c) State Integral Test. Test the convergence of series $\sum_{n=2}^{\infty} \frac{n^2}{4n^3-6}$. 7.5

6. (a) Define an absolutely convergent series. Prove that $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n^2}$ converges absolutely. 7.5

(b) State Root Test. Using Root test, discuss the convergence of $\sum_{n=1}^{\infty} \frac{5}{e^n}$ and $\sum_{n=2}^{\infty} \left(\frac{n-1}{3n}\right)^{2n}$. 7.5

(c) Prove that a non-negative series converges iff its sequence of partial sums is bounded above. 7.5