(b) Let $M(\alpha_1)$ be the Jukes-Cantor matrix with parameter ai and $M(\alpha_2)$ be the Jukes- Cantor matrix with parameter $\alpha_2$. Compute $M(\alpha_1)$ $M(\alpha_2)$ to show it has the form $M(\alpha_3)$. Express the formula for 013 in terms of $\alpha_1$ and $\alpha_2$.

(c) Calculate $d_{JC}(S_0, S_1)$ for the two 40-base sequences, showing all steps

$S_0$ : *CTAGGCTTACGATTACGAGGATCCAAATGGCACCAATGCT*

$S_1$ : *CTACGCTTACGACAACGAGGATCCGAATGGCACCATTGCT*



[This question paper contains 8 printed pages.]

**Your Roll No..............**

**Sr. No. of Question Paper : 1526** **I**

Unique Paper Code : 2353570010

Name of the Paper : Bio-Mathematics

Name of the Course : **Bachelor of Science**

Semester : V – DSE

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All the **six** questions are compulsory.

3. Attempt any **two** parts from each question.

4. **All** questions carry equal marks.

5. Use of non-programmable Scientific Calculator is allowed.

1. (a) Develop a continuous model for population growth using the given data, postulating a linear relationship between the population growth rate and the population size itself. Utilize this model to forecast the U. S. population in the year 3000.

| Year | Population |
|---|---|
| 1800 | 5.3 |
| 1810 | 7.2 |
| 1820 | 9.6 |
| 1830 | 12.9 |
| 1840 | 17.1 |
| 1850 | 23.2 |
| 1860 | 31.4 |

(b) Find the equilibrium states of the population growth model $P_{n+1} = \frac{2}{1+\frac{P_n}{K}} P_n$, where K is carrying capacity.

(c) Write the logistic growth model and find out the equilibrium points. Also, classify them as stable and unstable.



(b) Show that the iteration scheme

$$x_{n+1} = 1 - \mu x_n(1 - x_n)$$

has a stable fixed point $x_0 = 1$ for $\mu < 1$ and that $\mu = 1$ is a bifurcation point, where the fixed point is $x_0 = 1/\mu$ appears. Show that the period doubling occurs as soon as $\mu$ exceeds 3.

(c) Discuss the stability of limit cycle when $P_0$, $P_1$, $P_2$........ are the points on Poincare line and $P_n$ is at a distance of $s_n$ from $P_0$ and such that

$$s_{n+1} = f(s_n)$$

where $P_0$ is the fixed point.

6. (a) Write the transition matrices for Jukes-Cantor model and Kimura-2 model of base substitution stating the significance of each entry. In the Jukes Cantor Model of base substitution

(a) In what proportion of the sites will each base appear after one time step?

(b) What proportion of sites will have a base A in the ancestral sequence and a T in the descendent sequence one step later?

(b) Use Poincare'–Bendixson theorem to show that the system

$$\frac{dV}{dt} = f(V,O) = \frac{2}{3}V - \frac{V^2}{6} - \frac{VO}{1+V}$$

$$\frac{dO}{dt} = g(V,O) = \delta O\left(1 - \frac{O}{V}\right)$$

has a periodic solution for $0 < \delta < \frac{1}{12}$.

(c) Discuss the nature of the fixed points of the following system

$$\dot{x} = 5x + 2y,$$
$$\dot{y} = 2x + 2y.$$

and sketch the corresponding trajectories in the phase plane.

5. (a) Show that the non-linear conservative system

$$\dot{x} = y$$
$$\dot{y} = \mu \sin x - x$$

where $\mu \geq 0$ has one equilibrium point for $0 \leq \mu < 1$ and three for $\mu > 1$. Also discuss the nature of these equilibrium points.



2. (a) A drug whose elimination constant is r hours$^{\{-1\}}$ is first administered at 1 A. M. at a dosage of C μg/ml and in same dosage every three hour afterwards. What is the drugs concentration at 12:00 noon?

(b) A drug is available in doses of M mg, which raise the blood plasma concentration of an average adult by 10 mg $I^{-1}$. Once in the blood plasma, the drug concentration decays according to equation $\frac{dC(t)}{dt} = \frac{-C(t)}{\tau}$ with $\tau = 4$ hour. Health regulations require that the concentration never exceed 10 mg $I^{-1}$. What is shortest safe time interval at which doses can be given regularly? What is the minimum concentration if doses are given at this time interval?

(c) State of Law of Mass Action. Use it to derive a mathematical model governed by the intermediaries X and Y in trimolecular reaction:

$$A \to X$$
$$B + X \to Y + D$$
$$2X + Y \to 3X$$
$$X \to E$$

where A, B, D and E are initial and final products, and all rate constant are equal to 1.

3. (a) Discuss the direction of travel near the null clines of the following modified SIS model

$$\frac{dS}{dt} = -0.002\,SI + 0.1\,S,$$

$$\frac{dI}{dt} = 0.002\,SI - 0.4\,I.$$

Also sketch the phase plane diagram.

(b) Describe the SIR model with schematic representation and show that the epidemic will not take place if $S(0) < \frac{\beta}{\alpha}$.

(c) Stating all the assumptions of SIS model, describe the model. Show that

(i) For $N - \frac{\beta}{\alpha} < 0$, the number of infectives I(t) is declining and, thus, there is no epidemic.



(ii) If $N - \frac{\beta}{\alpha} > 0$, then the number of infectives is increasing with time and the number of susceptibles is decreasing with time, if $I(0) < N - \frac{\beta}{\alpha}$.

(iii) Find $\lim\limits_{t\to\infty} I(t)$ and $\lim\limits_{t\to\infty} S(t)$.

4. (a) For the limited growth prey-predator model

$$\frac{dV}{dt} = f(V,O) = \alpha V - \beta V^2 - \gamma OV$$

$$\frac{dO}{dt} = g(V,O) = -\delta O + \epsilon OV\ ,$$

where V denotes the population of prey and O denotes the population of predator. Calculate the equilibrium states of the system and discuss stability at the point $\left(\frac{\delta}{\epsilon}, \frac{\alpha}{\gamma} - \frac{\beta\delta}{\gamma\epsilon}\right)$.