

(b) Let  $M(\alpha_1)$  be the Jukes-Cantor matrix with parameter  $\alpha_1$  and  $M(\alpha_2)$  be the Jukes-Cantor matrix with parameter  $\alpha_2$ . Compute  $M(\alpha_1) M(\alpha_2)$  to show it has the form  $M(\alpha_3)$ . Express the formula for 013 in terms of  $\alpha_1$  and  $\alpha_2$ .

(c) Calculate  $d_{JC}(S_0, S_1)$  for the two 40-base sequences, showing all steps

$S_0 : CTAGGCTTACGATTACGAGGATCCAAATGGCACCAATGCT$

$S_1 : CTACGCTTACGACAACGAGGATCCGAATGGCACCATTGCT$

[This question paper contains 8 printed pages.]

**Your Roll No.....**

**Sr. No. of Question Paper : 1526**

**I**

Unique Paper Code : 2353570010

Name of the Paper : Bio-Mathematics

Name of the Course : **Bachelor of Science**

Semester : V – DSE

Duration : 3 Hours

Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable Scientific Calculator is allowed.

1. (a) Develop a continuous model for population growth using the given data, postulating a linear relationship between the population growth rate and the population size itself. Utilize this model to forecast the U. S. population in the year 3000.

| Year | Population |
|------|------------|
| 1800 | 5.3        |
| 1810 | 7.2        |
| 1820 | 9.6        |
| 1830 | 12.9       |
| 1840 | 17.1       |
| 1850 | 23.2       |
| 1860 | 31.4       |

- (b) Find the equilibrium states of the population growth

model  $P_{n+1} = \frac{2}{1 + \frac{P_n}{K}} P_n$ , where  $K$  is carrying capacity.

- (c) Write the logistic growth model and find out the equilibrium points. Also, classify them as stable and unstable.

- (b) Show that the iteration scheme

$$x_{n+1} = 1 - \mu x_n(1 - x_n)$$

has a stable fixed point  $x_0 = 1$  for  $\mu < 1$  and that  $\mu = 1$  is a bifurcation point, where the fixed point is  $x_0 = 1/\mu$  appears. Show that the period doubling occurs as soon as  $\mu$  exceeds 3.

- (c) Discuss the stability of limit cycle when  $P_0, P_1, P_2, \dots$  are the points on Poincare line and  $P_n$  is at a distance of  $s_n$  from  $P_0$  and such that

$$s_{n+1} = f(s_n)$$

where  $P_0$  is the fixed point.

6. (a) Write the transition matrices for Jukes-Cantor model and Kimura-2 model of base substitution stating the significance of each entry. In the Jukes Cantor Model of base substitution

- (a) In what proportion of the sites will each base appear after one time step?
- (b) What proportion of sites will have a base A in the ancestral sequence and a T in the descendent sequence one step later?

- (b) Use Poincare'-Bendixson theorem to show that the system

$$\frac{dV}{dt} = f(V, O) = \frac{2}{3}V - \frac{V^2}{6} - \frac{VO}{1+V}$$

$$\frac{dO}{dt} = g(V, O) = \delta O \left(1 - \frac{O}{V}\right)$$

has a periodic solution for  $0 < \delta < \frac{1}{12}$ .

- (c) Discuss the nature of the fixed points of the following system

$$\dot{x} = 5x + 2y,$$

$$\dot{y} = 2x + 2y.$$

and sketch the corresponding trajectories in the phase plane.

5. (a) Show that the non-linear conservative system

$$\dot{x} = y$$

$$\dot{y} = \mu \sin x - x$$

where  $\mu \geq 0$  has one equilibrium point for  $0 \leq \mu < 1$  and three for  $\mu > 1$ . Also discuss the nature of these equilibrium points.

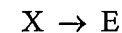
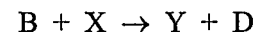
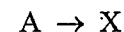
2. (a) A drug whose elimination constant is  $r$  hours<sup>(-1)</sup> is first administered at 1 A. M. at a dosage of  $C$   $\mu\text{g/ml}$  and in same dosage every three hour afterwards. What is the drugs concentration at 12:00 noon?

- (b) A drug is available in doses of  $M$  mg, which raise the blood plasma concentration of an average adult by  $10 \text{ mg l}^{-1}$ . Once in the blood plasma, the drug concentration decays according to equation

$$\frac{dC(t)}{dt} = \frac{-C(t)}{\tau} \text{ with } \tau = 4 \text{ hour. Health regulations}$$

require that the concentration never exceed  $10 \text{ mg l}^{-1}$ . What is shortest safe time interval at which doses can be given regularly? What is the minimum concentration if doses are given at this time interval?

- (c) State of Law of Mass Action. Use it to derive a mathematical model governed by the intermediaries  $X$  and  $Y$  in trimolecular reaction:



where A, B, D and E are initial and final products, and all rate constant are equal to 1.

3. (a) Discuss the direction of travel near the null clines of the following modified SIS model

$$\frac{dS}{dt} = -0.002 SI + 0.1 S,$$

$$\frac{dI}{dt} = 0.002 SI - 0.4 I.$$

Also sketch the phase plane diagram.

- (b) Describe the SIR model with schematic representation and show that the epidemic will not

take place if  $S(0) < \frac{\beta}{\alpha}$ .

- (c) Stating all the assumptions of SIS model, describe the model. Show that

- (i) For  $N - \frac{\beta}{\alpha} < 0$ , the number of infectives

$I(t)$  is declining and, thus, there is no epidemic.

- (ii) If  $N - \frac{\beta}{\alpha} > 0$ , then the number of infectives is increasing with time and the number of susceptibles is decreasing with

time, if  $I(0) < N - \frac{\beta}{\alpha}$ .

- (iii) Find  $\lim_{t \rightarrow \infty} I(t)$  and  $\lim_{t \rightarrow \infty} S(t)$ .

4. (a) For the limited growth prey-predator model

$$\frac{dV}{dt} = f(V, O) = \alpha V - \beta V^2 - \gamma OV$$

$$\frac{dO}{dt} = g(V, O) = -\delta O + \epsilon OV,$$

where V denotes the population of prey and O denotes the population of predator. Calculate the equilibrium states of the system and discuss

stability at the point  $\left( \frac{\delta}{\epsilon}, \frac{\alpha}{\gamma} - \frac{\beta\delta}{\gamma\epsilon} \right)$ .