This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2232**

Unique Paper Code : **2352572301**

Name of the Paper : **Differential Equations**

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor**

Type of the Paper : **DSC-B2**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **90**

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

Attempt *all* questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of simple Calculator is allowed.

1. (*a*) Find the differential equation from the relation $y = a \sin x + b \cos x + x \sin x$, where a and b are arbitrary constants. 7.5

(*b*) Solve the differential equation : 7.5

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$$

(*c*) Show that $y_1 = e^x$ and $y_2 = xe^x$ are the solutions of the differential equation : 7.5

$$y'' - 2y' + y = 0.$$

Also, verify that y_1 and y_2 are linearly independent.

P.T.O.

2. (*a*) Solve the following homogeneous differential equation : 7.5

$$x\frac{dy}{dx} = x + y.$$

(*b*) Find the integrating factor of the following differential equation and solve it : 7.5

$$(x^2 + y^2 + x)dx + xy\,dy = 0.$$

(*c*) Find the orthogonal trajectories of the family of curves $y^2 = 2x^2 + c$, where c is a real parameter. 7.5

3. (*a*) Find the general solution of the differential equation by using the method of undetermined coefficients : 7.5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x.$$

(*b*) Solve the initial value problem : 7.5

$$y'' + 3y' - 10y = 0,\ y(0) = 4,\ y'(0) = -9$$

(*c*) Find the general solution of the differential equation by using the method of variation of parameter : 7.5

$$\frac{d^2y}{dx^2} + y = \tan x.$$

4. (*a*) Find the general solution of the given differential equation : 7.5

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3.$$

(*b*) Write the general solution of the 10th-order homogeneous differential equation whose auxiliary equation has the following roots :

$$0,\ 4,\ 3,\ 3,\ 3,\ -3,\ 1 + 4i,\ 1 - 4i,\ 1 + i,\ 1 - i.$$

Verify that e^x and e^{2x} are solutions to the homogeneous differential equation :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0.$$

Show that a linear combination of e^x and e^{2x} is also a solution. 7.5

(c) Find the general solution of the linear system : 7.5

$$\frac{dx}{dt}+\frac{dy}{dt}-2x-4y=e^t,$$

$$\frac{dx}{dt}+\frac{dy}{dt}-y=e^{4t}.$$

5. (a) Eliminate the arbitrary function f from the question : 7.5

$$z = xy + f(x^2 + y^2),$$

and obtain the partial differential equation. Also, specify the order.

(b) Find the solution of the Cauchy problem for the first order PDE : 7.5

$$3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0, \quad u(x,0)=\sin x.$$

(c) Reduce the equation : 7.5

$$u_x+u_y=u,$$

to canonical form and obtain the general solution.

6. (a) Use the Method of Characteristics to find the general solution of the equation : 7.5

$$x^2u_x + y^2u_y = -u^2.$$

(b) Apply the method of separation of variables $u(x, y) = X(x)Y(y)$ to solve : 7.5

$$u_x - u_y = -u, \qquad u(0, y) = 6e^{-2y}.$$

(c) Classify the partial differential equation as elliptic, parabolic or hyperbolic :

$$u_{xx} = x^2u_{yy}, \; x \neq 0.$$

Also, reduce it to canonical form. 7.5