This question paper contains **3** printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **1565**

Unique Paper Code : **2352571101**

Name of the Paper : **DSC : Topics in Calculus**

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting *two* parts from each question.

All questions carry equal marks.

1. (*a*) Show that the function $f(x) = |x| + |x - 1|$ is not differentiable at $x = 0$ and $x = 1$.

(*b*) Find the *n*th differential coefficients of $\sin^5 x \cos^3 x$.

(*c*) If $u = \sin^{-1} \dfrac{x^2 + y^2}{x + y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.

2. (*a*) Examine the continuity of the function $f(x) = \dfrac{\sqrt{1 - \cos 2x}}{x}$, $x \neq 0$; $f(0) = 0$, $x = 0$.

In case of discontiunity, state the kind of discontinuity.

(*b*) If $y = \left[x + \sqrt{1 + x^2}\right]^m$, find $y_n(0)$.

(*c*) If $= e^{xyz}$, show that $\dfrac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2h^2z^2)e^{xyz}$.

P.T.O.

3. (*a*) Evaluate : $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$.

(*b*) Show that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots\ldots\ldots\ldots\ldots + \frac{(-1)^{n-2}x^{n-1}}{n-1} + \frac{(-1)^{n-1}x^n}{n(1+\theta x)^n},\ 0 < \theta < 1.$$

(*c*) Verify whether Rolle's theorem is applicable to the function $f(x) = 2 + (x-1)^{\frac{2}{3}}$ in the interval [0, 2] or not.

4. (*a*) Expand $e^{\sin x}$ as far as the term containing x^4.

(*b*) If in Cauchy's mean value theorem, $f(x) = \frac{1}{x^2}$, and $g(x) = \frac{1}{x}$, then show that c is the harmonic mean of a and b.

(*c*) Find a, b, c so that $\lim_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$.

5. (*a*) Find all the asymptotes of the curve :

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0.$$

(*b*) Find the range of values of x in which the curve

$$y = 3x^4 - 4x^3 + 1$$

is concave upwards or downwards. Also find its points of inflexion.

(*c*) Find the reduction formula for

$$\int \sin^n x\,dx.$$

Hence, evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x\,dx$.

6. (*a*) Determine the position and nature of the double points on the curve :

$$x^3 + x^2 + y^2 - x - 4y + 3 = 0.$$

(*b*) Trace the curve $y = x(x^2 - 1)$.

(*c*) Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx$. Hence evaluate :

$$\int_0^{\frac{\pi}{2}} \sin^4 x \cos^5 x \, dx.$$