(ii) Obtain the $V(\hat{\theta}_1), V(\hat{\theta}_2)$ and $cov(\hat{\theta}_1, \hat{\theta}_2)$

(7,7)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 3407 **I**

Unique Paper Code : 2374000006

Name of the Paper : Introduction to Statistical Linear Models

Name of the Course : **GE under NEP-UGCF**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. **Question 1** is compulsory.
3. Attempt **six** questions in all.
4. Use of a simple scientific calculator is allowed.

1. Attempt **any five** parts

(a) Explain the concept of R-Squared Statistic (R^2) and Adjusted R-Squared Statistic (Adj-R^2).

(b) Write the statement of Gauss Markov Theorem for full rank case.

(c) Define a statistical linear model. Differentiate between fixed effects, random effects, and mixed effects linear models.

(d) Consider the following model with the usual assumptions: $E(y_1) = 2\beta_1 + \beta_2$, $E(y_2) = \beta_1 - \beta_2$, $E(y_3) = \beta_1 + \alpha\beta_2$. Determine α so that the BLUES of β_1, β_2 are uncorrelated.

(e) Explain residual analysis and discuss its uses.

$-0.28xz$. What is the distribution of $0.29x^2 - 0.86xy + 0.64y^2 + 0.07z^2 + 0.28xz - 0.42yz$? Are these two quadratic forms independent?

(b) Suppose we postulate the model $E(Y) = \beta_0 + \beta_1 x$ but the model $E(Y) = \beta_0 + \beta_1 x + \beta_{11} x^2 + \beta_{111} x^3$ is actually the true response function, unknown to us. If we use observations of Y at $x = -2, -1, 0, 1, 2$ to estimate β_0 and β_1 in the postulated model, what biases will be introduced? (7,7)

7. (a) Define multiple regression model and polynomial regression model.

(b) Let X_i, Y_i, Z_i, i=1,2,3 be 9 independent observations with common variance σ^2. If $E(X_i) = \theta_1$, $E(Y_i) = \theta_2$, $E(Z_i) = \theta_1 - \theta_2$, $i = 1, 2, 3$.

(i) Find the $BLUE_S$ of θ_1, θ_2 and $\theta_1 + \theta_2$

(iii) At 0.05 level of significance, can one conclude that the classes of factor A have different effects? Why? (Use $F_{0.05[2,23]}$=3.42, $F_{0.05[7,14]}$=2.76, $F_{0.05[2,23]}$=3.74, $F_{0.005[2,7]}$=4.74) (6,8)

5. Let $\underset{\sim}{Y}$= $(Y_1, Y_2,..., Y_n)'$ be a vector of n independent standard normal variates. Let Q_1= =$\underset{\sim}{Y}'A_1\underset{\sim}{Y}$and $Q_2 = \underset{\sim}{Y}'A_2\underset{\sim}{Y}$ be distributed as χ^2 with ni and n2 degrees of freedom respectively. Prove that the necessary and sufficient condition for Q_1 and Q_2 to be independently distributed is A_1A_2 = O. (14)

6. (a) Let x, y, and z denote three independent standard normal variates. Stating appropriate theorem, find the distribution of homogenous equation of second degree, $0.71x^2 + 0.86xy + 0.36y^2+0.93z^2 +0.42yz$

(f) An advertising company is investigating which type of advertisement (ad) increases the sale most. It chooses 16 cities, 4 selling agents and 4 types of ads. In each city an ad is assigned randomly to an agent for a period of four weeks and the figures on sales are used to find the effects of the ads. Give the name of the experimental design used in this case and also write the ANOVA table of the design. (5 x 4)

2. Consider the simple linear regression model (SLRM) $y = \beta_0 + \beta_1 x + \varepsilon$ given that $E(\varepsilon)$ = 0, $V(\varepsilon)$ = $\sigma^2, \varepsilon's$ are uncorrelated.

(i) Obtain the least squares estimates of β_0 and β_1 and show that they are unbiased.

(ii) Obtain the variance of $\hat{\beta}_0$ and $\hat{\beta}_1$.

(iii) Find the confidence intervals of β_0, β_1 and σ^2. (14)

3. For the one-way classified fixed effect model, $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$; $i = 1,2,..., k$; $j = 1,2, ..., n$, where the symbols have their usual meaning.

(i) Obtain the estimates of the parameters μ and $\alpha_i's$.

(ii) Derive the expectations of the various sum of squares.

(iii) Write the ANOVA Table. (14)

4. (a) For the multiple linear regression model derive the test statistic for testing the significance of individual regression coefficients.

(b) Fill the missing entries of the partially completed two-way ANOVA table:

Source of Variation	Degrees of freedom	Sum of Squares	Mean Squares	Variance Ratio
Factor A		231.5054	115.7527	
Factor B		98.5	14.0714	
Error		573.75		
Total	23			

(i) How many classes/levels of factor A are being compared?

(ii) How many observations are analyzed?