Prove that if X follows F- distribution with (m, n) d. f.and Y follows the F- distribution with (n, m) d. f., then for every a > 0, P (X $\leq$ a) + P(Y $\leq$ 1/a) = 1. (7,8)

7. (a) Let X_1, X_2,..., X_n be a random sample from N (μ,σ^2) and $\overline{X}$ and s^2 the sample mean and sum of the squares of the deviations from the mean respectively. Let X'~N (μ,σ^2) and assume that X_1, X_2,..., X_n, X' are independent, obtain the sampling distribution of $U = \frac{(X' - \overline{X})}{s} \cdot \sqrt{\frac{n(n-1)}{(n+1)}}$.

(b) Define F statistic. In one sample of 8 observations, the sum of squares of deviations of the observations from the sample mean is 84.4 and in the other sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level of significance. Given that $F_{0.05}$ [7,9] = 3.29. (7,8)

8. Write short notes on any three of the following:

(i) Null hypothesis and level of significance.

(ii) Relation between chi-square and F distributions.

(iii) Type I and Type II errors.

(iv) Mean deviation about mean for the t-distribution with n d.f. (5,5,5)

(1000)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3074 **I**

Unique Paper Code : 2374002001

Name of the Paper : Sampling Distributions

Name of the Course : **GE 3(a) Sampling Distributions**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt **any Six** questions in all.

1. (a) Discuss the test of significance for difference of proportions for large samples.

(b) Explain the term standard error and sampling distribution. Show that in a series of n independent trials with constant probability p of success, the

P.T.O.

standard error of the proportion of success is $\left(\frac{pq}{n}\right)$, where q = 1-p. (7,8)

2. (a) A sample of 15 values shows that the s.d. is 6.4. Is this compatible with the hypothesis that the sample is from a normal population with s.d. 5 at 5% level of significance? Justify your answer.

$\left[\chi^2_{0.05} \; for \; 13 \;\; d.f. = 22.3\right]$

(b) Show that chi-square distribution tends to normal distribution for large n. (7,8)

3. (a) Define Fisher's t statistic. Find mean, variance and β_1 of t distribution with n d.f..

(b) Show that the probability curve of F distribution is positively skewed.

If X is a F-variate with 2 and n d.f.. Show that $P(F \geq k) = (1+2k/n)^{-n/2}$. (7,8)

4. (a) Derive the χ^2 test of goodness of fit of a theoretical distribution to an observed frequency distribution with assumptions. The following table gives the number of aircraft accidents that occurred

during the seven days of the week. Find whether the accidents are uniformly distributed over the week. (χ^2 0.05 for 6 d.f. =12.592)

Days:	Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun.
No. of accidents	14	18	12	11	15	14	14

(b) Define Chi-square variate with 1 d.f. Derive p.d.f. of a Chi-square distribution with n d.f.. (7,8)

5. (a) The average hourly wage of a sample of 150 workers in a plant 'A' was Rs. 2.56 with a standard deviation of Rs. 1.08. The average hourly wage of a sample of 200 workers in plant 'B' was Rs. 2.87 with standard deviation of Rs. 1.28. Can an applicant safely assume that the hourly wages paid by plant 'B' are higher than those paid by plant 'A'?

(b) For the t-distribution with n d. f., prove that

$$\mu_{2r} = \frac{n(2r-1)}{(n-2r)}\mu_{2r-2} \;, \; n > 2r. \qquad (7,8)$$

6. (a) Obtain the mode of F distribution with ni and n2 degrees of freedom, also show that it lies between 0 and 1.

(b) If X has an F distribution with (n_1 n_2) degrees of freedom, find the distribution of 1/X .

P.T.O.