This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **3061**

Unique Paper Code : **2354002002**

Name of the Paper : **Lattices and Number Theory**

Name of the Course : **GE**

Semester : **III**

Programme : **COMMON PROG GROUP (NEP-UGCF 2022)**

Duration : **3 Hours** Maximum Marks : **90**

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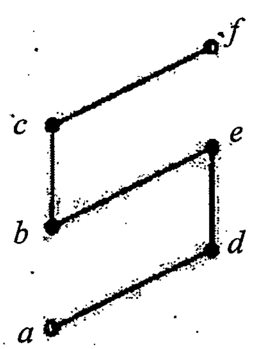
Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

Use of simple calculator is allowed.

1. (*a*) Define a totally ordered set. Let P = {1, 2, 3, 4, 6, 9, 12, 18, 36} and $x < y$ in P if x divides y. Prove that P is a partial ordered set under the order relation <. Is P totally ordered set ? Justify.

(*b*) Define top and bottom elements of a partial order set. List all the maximal, minimal, all the lower and upper bounds along with top and botton element of the following partial ordered set :

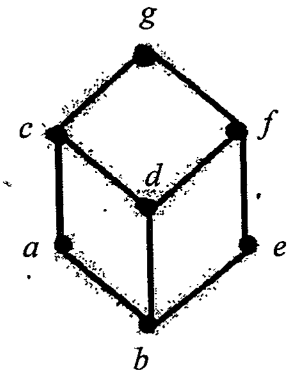


P.T.O.

(c) Draw the Hesse Diagrams for the partial order sets $2 \cup 4$ and 2×2 where n denote the chain :

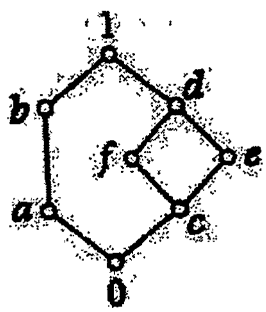
$$0 < 1 < 2 < 3 < \ldots < n - 1$$

2. (a) Define sublattice of a lattice L. Consider the following lattice L :



Find three sublattices of the above lattice. Deduce that the union of two sublattices is not a sublattice of L.

(b) Write distributive laws in a lattice L. State the criterion for a lattice to be distributive. Is the following lattice distributive ? Justify.



(c) Find minimal form of the polynomial $p = xy + \bar{y}$ by using :

(i) Karnaugh maps

(ii) Quine-McCluskey method.

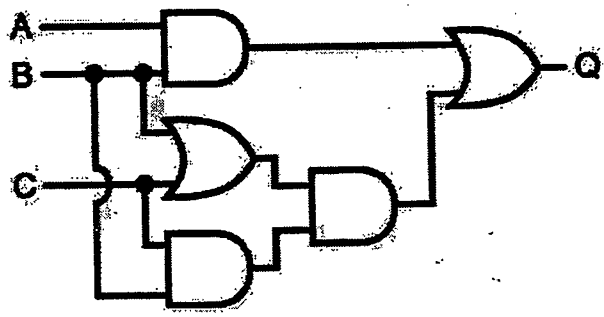
3. (a) State the Duality Principle in a partial orderd set. Find the dual of :

   (i) $f = xy + yz + zx$

   (ii) $f = \bar{x}y + \bar{y}x$.

   (b) Find the DN and CN form of $p = x + xy$.

   (c) Define Boolean Algebra. Represent the following circuit diagram in the form of Boolean expression :



4. (a) State Division Algorithm. Use the Euclidean Algorithm to obtain integers x and y satisfying the equation :

$$\gcd(24,138) = 24x + 138y.$$

   (b) State Wilson's theorem and prove that :

$$1^2.3^2.5^2....(p-2)^2 \equiv (-1)^{\frac{p+1}{2}} (\text{mod } p).$$

   (c) State Chinese Remainder Theorem and obtain eight incongruent solutions of the linear congruence :

$$3x + 4y \equiv 5 \,(\text{mod } 8).$$

5. (a) Define Euler's Phi-Function. Use Euler's theorem to establish that for any integer 'a' :

$$a^{37} \equiv a(\text{mod } 1729).$$

(*b*) Define primitive root of a integer n with example. If $F_n = 2^{2^n}$, $n > 1$ is a prime, then prove that 2 is not a primitive root of F_n.

(*c*) Find the order of the integer 3 modulo 19.

6. (*a*) Solve the following quadratic congruence :

$$3x^2 + 9x + 7 \equiv 0 \pmod{13}.$$

(*b*) Define quadratic residue of an odd prime p. Find all the quadratic residues of 17.

(*c*) Define Legendre's symbol. Find the value of the following Legendre symbols :

(*i*) (19/23)

(*ii*) (20/31).