$$\frac{dx}{dt} = 3x + 4y \ , \qquad \frac{dy}{dt} = 2x + y. \qquad (7.5)$$

Write the general solution.

5. (a) Find the solution of the equation $xu_x + yu_y = xe^{-u}$ with Cauchy data $u = 0$ on $y = x^2$. (7.5)

(b) Find the general solution of the equation

$$yz\,u_x - xz\,u_y + xy(x^2 + y^2)u_z = 0. \qquad (7.5)$$

(c) Reduce the following differential equation into canonical form and find thegeneral solution

$$u_x + u_y = u. \qquad (7.5)$$

6. (a) Using V = In u and $V(x, y) = f(x) + g(y)$, show that the solution of the Cauchy problem

$$y^2 {u_x}^2 + x^2 {u_y}^2 = (xyu)^2; \ u(x,0) = e^{x^2}$$

is$u(x,y) = \exp\left(x^2 + i\frac{\sqrt{3}}{2}y^2\right)$. (7.5)

(b) Find the solution of the initial value systems

$$u_t + uu_x = e^{-x}\,v, \qquad v_t - av_x = 0,$$

with $u(x, 0) = x$ and $v(x, 0) = e^x$. (7.5)

(c) Obtain the general solution of

$$u_{xx} - 4u_{xy} + 4u_{yy} = e^y. \qquad (7.5)$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3060 **I**

Unique Paper Code : 2354002001

Name of the Paper : Differential Equations

Name of the Course : **COMMON PROG GROUPS (Generic Elective)**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting two parts from each question.
3. Both parts of the question to be attempted together.
4. All questions carry equal marks.
5. The use of a calculator is not allowed.

1. (a) Determine the values of m for which the function $f(x) = e^{mx}$ is a solution of the differential equation

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 12y = 0. \qquad (7.5)$$

(b) Solve the initial-value problem

$(2x \cos y + 3x^2y)\, dx + (x^3 - x^2 \sin y - y)\, dy = 0;$
$y(0) = 2.$ (7.5)

(c) Solve the differential equation

$$\frac{dy}{dx} = \frac{x^2y}{x^3 + y^3}. \qquad (7.5)$$

2. (a) Solve the differential equation

$$x\frac{dy}{dx} + y = -2x^6y^4. \qquad (7.5)$$

(b) Find an integrating factor and hence solve the following equation

$(x^2 + y^2 + x)\, dx + xy\, dy = 0.$ (7.5)

(c) Find the orthogonal trajectories of the family of curves $y = ax^2$ where a is the parameter. (7.5)

3. (a) Find the solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0;\ y(0) = -3,\ y'(0) = -1. \qquad (7.5)$$

(b) Use the method of undetermined coefficients to find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \cos 4x. \qquad (7.5)$$

(c) Use the method of variation of parameters to find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = \cot x. \qquad (7.5)$$

4. (a) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6. \qquad (7.5)$$

(b) Solve the linear system

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$$
$$\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}. \qquad (7.5)$$

(c) Show that $x = 2e^{5t},\ y = e^{5t}$ and $x = e^{-t},\ y = -e^{-t}$ are two linearly independent solutions on every interval $a \le t \le b$ of the homogeneous linear system